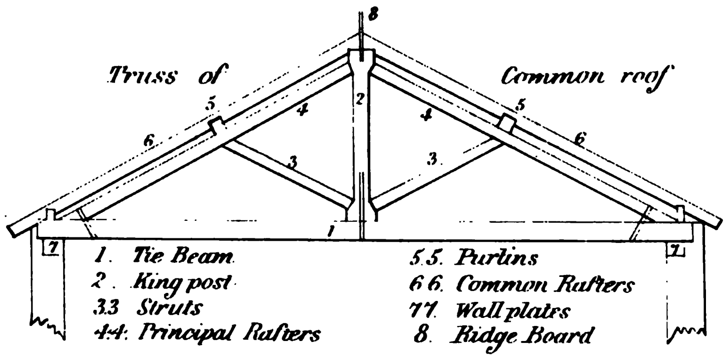
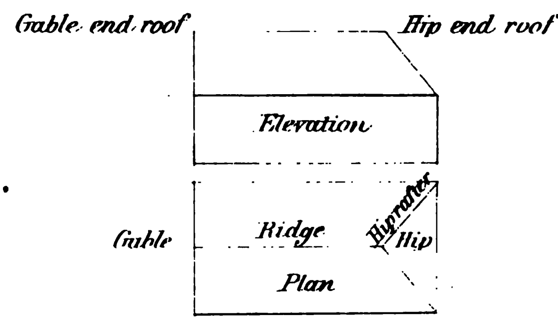
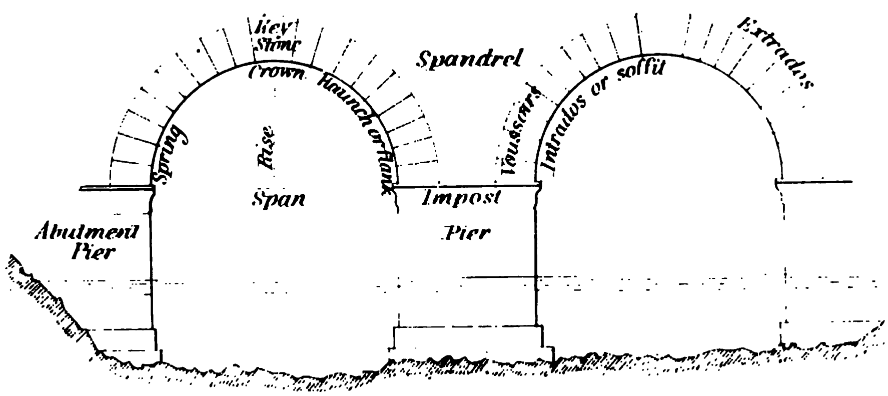
FRONTISPIECE.

Technical terms often met with, in Geometrical Drawing.







Arches

FIRST PRACTICAL LINES

IN

GEOMETRICAL DRAWING.

CONTAINING

A COPIOUS SERIES OF EXAMPLES AND PROBLEMS

IN

PRACTICAL GEOMETRY,

USE OF MATHEMATICAL INSTRUMENTS,

CONSTRUCTION OF SCALES, DESCRIPTIVE GEOMETRY,

ORTHOGRAPHIC AND HORIZONTAL PROJECTIONS,

THEORY OF SHADOWS,

ISOMETRICAL DRAWING, AND PERSPECTIVE.

THE WHOLE FOUNDED ON QUESTIONS GIVEN AT THE DIFFERENT MILITARY AND OTHER COMPETITIVE EXAMINATIONS,

AND ILLUSTRATED WITH UPWARDS OF 300 DIAGRAMS.

BY

J. F. H. DE RHEIMS, F.C.S., ETC.

FOR MANY YEARS PROFESSOR OF FORTIFICATION, FREEHAND AND GEOMETRICAL DRAWING, NATURAL PHILOSOPHY, AND CHEMISTRY, IN MOST OF THE OLDEST AND PRINCIPAL MILITARY ESTABLISHMENTS IN THE NEIGHBOURHOOD OF LONDON AND IN WOOLWICH.

WILLIAMS AND NORGATE,

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PREFACE.

In accordance to the wishes of several of the proprietors of establishments where I have the honour to attend, as well as to meet the desire manifested by many of my pupils of having some kind of class-book to guide them in the prosecution of their drawing studies, and which should, at the same time, combine moderate cost, plain and concise rules, and a copious amount of examples and of questions to be solved, I had prepared a work not only giving the solution of every question which had hitherto been set at the divers military and other examinations, but treating likewise on every subject connected with the science, and illustrated with nearly 800 diagrams. When completed, I found that I had miscalculated—that not only its publication would be too expensive, and thereby defeat one of its principal objects, but that its bulk would acquire dimensions too extensive for such an elementary work as I had in view. I felt, therefore, compelled to condense it to its present size, to expunge entirely some of its technical subjects, such as Mechanical and Architectural Drawing, Fortification, &c.; to substitute rough lithographs for the finished engraved diagrams with which I had intended to illustrate it, and to confine myself to the publication of that which, judging from past examinations, I considered most absolutely necessary.

However humble may be its object, a book of this kind cannot expect to escape criticism; and as a foreigner, I feel painfully conscious that its language is particularly open to it. On this point, however, as a literary work is not intended to be produced, I throw myself on the indulgence and generosity of my readers.

Other dangers likewise commonly attend the produc-

tion of such a book, which, to a certain extent, must necessarily be a compilation. Let the author follow the beaten path—repeat what has been explained before him —and he lays himself open to an accusation of plagiarism. On the other hand, let him emit new ideas, attempt new methods, and although he obtains the same correct results, he exposes his book to be rejected by those who, unaccustomed to the proposed alterations, often consider as "heresies" any deviations from their own familiar process. Disregarding these considerations, and without following invariably any particular method, I have selected what my experience induced me to consider as most useful. When I have found it necessary, I have not scrupled to consult works treating especially upon the branches having reference to the subjects under explanation. In most cases, however, I have endeavoured, when practicable, to simplify complicated methods. If in some of these I have failed, I am open to correction which will be thankfully received.

Some of my observations will perhaps appear scanty, because, wishing as much as possible to reduce the solution of individual subjects to general rules, I have seldom repeated the explanations given at the beginning of the chapter to which they refer, and which, to attentive and intelligent pupils should prove amply sufficient.

I beg, finally, to repeat that this small treatise is not intended to act as a substitute for, but merely as an adjunct to masters, and that its condensation was planned as a means of obtaining at the same time a cheap and useful text-book, acceptable both to tutors and pupils. That this may be the case, and meet with approval, is my sincere wish.

H. DE RHEIMS.

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CONTENTS.

PRACTICAL LINES

IN

GEOMETRICAL DRAWING.

As in the following pages my intention is to be as concise as possible, and to avoid everything not absolutely necessary, I will confine myself to those parts of Geometrical drawing, to those descriptions of instruments, and to those observations which an experience of twenty-five years has taught me to be the most required by pupils.

Without entering into the description of those implements usually contained in a box of mathematical instruments, the use of which may almost be learnt intuitively, I will merely observe that "good work depends in a great measure on good tools," and that too much care cannot be bestowed on our instruments, especially with respect to cleanliness, sharpness of points, smoothness of edges, and accuracy of scales, rulers, &c. The following rules, therefore, should always be borne in mind.

1. In geometrical drawing, all lines should at first be lightly drawn with a pencil moderately hard—HH.

2. The indian rubber should be used as sparingly as possible, and not until some time after the diagram has been inked in.

3. In inking in the different lines should, as much as possible, be drawn according to their order of construction, but when there are tangents to curves, begin by inking in the curves.

4. The beauty of a line consists in being throughout of an even and uniform thickness.

5. The indian ink should be carefully rubbed, very dark, and free from grit.

6. In drawing circles, the compass should be held very lightly, and in a vertical position, yet so as to avoid making holes in the paper.

7. In geometrical constructions, the nature of the lines usually determine their meaning, thus:—

Thin lines, ———, are generally employed for data; thick lines, ———, for the representation of results, or quaesita, and dotted lines for lines of construction.

In intricate problems the nature of the lines of construction vary with the progress of the diagram, thus, in the first period, the lines are usually dotted, ..., or barred, --------; in the second period, alternately dot and bar .-.-.-.-.; in the third construction alternately two dots and bar, ..—..—..—, and so on. Dotted lines are also used in solids to determine the position of lines unseen but necessary to the conception of the diagram, also to determine shadows, &c. As these lines are conventional, practice will teach what kind of lines should be used in any case.

8. When lines meet, the angle formed by them should be sharply defined.

9. For the sake of distinction original or given points are often surrounded by a small circle ⊙ to distinguish them from resulting points.

10. In determining points by the intersection of curves, the arcs should, if possible, cross nearly at right angles to each other, never at an angle less than 60°.

The student should begin by drawing a series of pencil lines and then inking them in with lines of different thicknesses, and do the same with a series of concentric circles of different radii; before inking in, experience will teach

him the value of previously trying his pen on a piece of waste paper.

He should also practise the imitation of printed letters. Plain Roman capital and small letters, and Italic capital and small letters are those generally used in military drawing. Any well printed book will furnish him with the most useful and best examples.

Besides different kinds of compasses, ruling pens, &c., good cases of instruments are usually provided with a protractor and a sector. Military boxes contain also a set of Marquois scales.

PROTRACTOR.

A protractor sometimes consists of a horn or brass semicircle, divided into 180 equal parts, named degrees, and is used to determine the value of angles.

The best protractors, however, are made either of boxwood or ivory, and are in the shape of parallelograms, 6 inches long by 1.5 broad (*vide* Fig. 1, Plate I.); they contain, besides their graduated edge,—

Two scales of chords of different sizes, the smallest marked C and the largest Cho, and whose purposes are the same as those of the graduated edge of the protractor, viz., to measure angles.

A scale of inches, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{1}{4}$, $\frac{1}{8}$, of an inch duodecimally divided.

A series of plane scales divided in the ratios of 20, 25, 30, 35, 40, 45, 50, 55 and 60 units to one inch.

Diagonal scales divided in the ratios of 200 and 400 units to one inch.

A plotting scale, either divided at the rate of 40 units to one inch, or, in military protractors, at the rate of one mile to 4 inches, or 440 yards to one inch.

N.B.—The Student is expected to follow, with the help of his own instruments, the directions here given.

To construct an angle by the protractor.

Draw a base line, BC, and determine on it the point at which the angle is to be constructed; place the centre A of the protractor on the given point, so that its lower edge coincides with the given base; determine the required angle with a finely pointed pencil, and through that obtained point draw a line from the original one.

PRACTICE.

1. On a base AB, two inches long, construct a triangle, so that the angle ABC equals 65°, and the angle BAC equals 47°. *Vide* Fig. 2. Place the protractor at the left extremity A of the given line, and determine the angle 47° inclined towards the right; remove the protractor to the right extremity B, and determine likewise the angle B, 65°, inclined towards the left.

N.B.—Remember that lines at angles less than 90° incline towards each other.

2. Construct a triangle ABC whose base AB = 1.7 inches, whose angle ABC = 42°, and whose angle BAC = 57°.

3. On a base AB, 1.5 inches long, construct a triangle, and let the angle BAC = 105°, and the angle ABC = 37°.

If with any radius we describe a circle, we shall find that the radius used as a chord will be contained exactly six times within the circumference; as a circumference is divided into 360°, it follows that the radius equals the chord of $\frac{1}{6}$ of the area, or 60°. The construction of the scales of chords, and of most sectoral lines, depend on that fact.

To construct a scale of chords.

Draw two lines, AB, BC, of any length, and at right angles to each other; from B as a centre describe with any radius the arc AC: respectively from A and C as centres with radius AB cut AC in D and E.

The triangle ABD will equal $\frac{1}{3}$ of 90°, or 30°.

The triangle ABE will equal $\frac{2}{3}$, or 60°.

Trisect (by trials) each of these three sectors, and mark them 1, 2, 3, 4, 5, 6, 7, 8 and 9. The quadrand ADEC will be divided into 9 equal parts, each of which will be worth 10 degrees.

Divide therefore each of these nine parts into 10 subdivisions, to show single degrees (for the present purpose this may be omitted).

Draw the chord AC, and from A as a centre transfer the divided distance A1, A2, and A8 on the circumference (Fig. 3), to the chord AC, which number 10, 20, 30—80, &c.

Each of these divisions will equal 10 degrees, and the line AC will become a scale of chords of 90°.

To determine any angle, say 40°, *by the scale of chords* (*see* Fig. 4).

Draw any line long enough to contain the length A—60 taken on the scale of chords (Fig. 3), and with that length as a radius describe the arc BC; take in the compass the length A 40° (the value of the angle required) and adapt it from B to D (Fig. 4); the distance BD is the chord of the angle of 40°.

To obtain any other angle, say 70°. (Fig. 4.)

On the same arc, or on one similarly obtained, with the same radius, adapt the length A—70 obtained on the same scale of chords, from B to G.

Angles as great as 90° can be obtained in a similar manner.

When they exceed 90° we must repeat the operation until the value of the angle is obtained: thus, let an angle of 100° be required. Determine the angle of 50° BE and repeat it on EF: BF will be the chord of an angle of 100°. (Fig. 4.)

Practice.

1. By scale of chords, determine angles of 32°, 47°, 76°, 92° and 117°, and 153°, and prove those angles by the graduated edge of the protractor.

2. Construct a triangle having two of its sides 4.62 and 3.47 inches long respectively and the included angle = 48°: figure the remaining angles and the length of the third side.

3. Construct a triangle ABC, of which the sides AB, BC and CA are respectively 1.72, 2.9 and 2.68 inches: determine the number of degrees contained in each of its angles.

Use of Scales.

In order to understand the use of scales, it is desirable that we should be acquainted with their construction.

The scales generally in use are either the plain scale or the diagonal scale.

Plain scales contain primary and secondary divisions.

The primary or larger divisions represent whole numbers or multiples of 10 units.

The secondary divisions are subdivided into as many units or fractional parts as are contained in a primary division.

To draw a plain scale, say of 10 units to 1 inch and 60 units in length. (*Vide* Fig. 5.)

Draw a line 6 inches long and divide it into 6 equal parts, (each of these parts will equal 10 units).

Subdivide the first left hand division into 10 equal parts. (Euclid 2, VI. book.) (*Vide* Practical Geometry, problem 7.)

Draw a second line (thick) at $\frac{4}{60}$th of an inch below and parallel to the first, and number it as in the given example.

So far the scale of 10 units to one inch will be completed. All plain scales are constructed according to the same

principle, and vary from each other only in their proportions, which are determined by the index accompanying them, thus the scales preceded by the numbers 60, 50, 40, 30, 20, &c. signify that the space of one inch is subdivided in the ratios of 60, 50, 40, 30, 20 equal parts, &c. The mode of division remaining the same in every one.

PRACTICE.

1. Draw a line A-B 6 inches long, and upon it, from A as a common starting point, determine by the plane scale and number the distances of

$$\tfrac{147}{60} \quad \tfrac{82}{48} \quad \tfrac{176}{40} \quad \tfrac{48}{35} \quad \tfrac{77}{30} \quad \text{and} \quad \tfrac{55}{20}$$

It is clear that these scales will also enable us to obtain vulgar fractional parts of the inch—as, should we require

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{8}$ $\frac{1}{9}$ $\frac{1}{10}$ $\frac{1}{11}$ $\frac{1}{12}$ $\frac{1}{14}$ $\frac{1}{15}$ $\frac{1}{16}$ $\frac{1}{18}$ of an inch.

The scale of $\frac{20}{10}=2$ of $\frac{30}{10}=3$ of $\frac{40}{10}=4$ of $\frac{50}{10}=5$ of $\frac{60}{10}=6$ of $\frac{35}{5}=7$ of $\frac{40}{5}=8$ of $\frac{45}{5}=9$ of $\frac{50}{5}$ or $\frac{20}{2}=10$ of $\frac{55}{5}=11$ of $\frac{60}{5}=12$ of $\frac{35}{2.5}=14$ of $\frac{30}{2.}=15$, of $\frac{40}{2.5}=16$ of $\frac{45}{2.5}=18$, &c.

PRACTICE.

1. Find $1.\frac{2}{7}$ and $2\frac{1}{6}$ inches on your scale.
   Now $1\frac{2}{7} = \frac{9}{7} = \frac{45}{35}$ and $2\frac{1}{6} = \frac{13}{6} = \frac{130}{60}$
2. Find $1\frac{1}{8}$ $2.\frac{7}{9}$, $3\frac{4}{11}$, $\frac{13}{9}$ $\frac{7}{5}$ and $\frac{10}{12}$ of an inch.

The diagonal scales accompanying the protractors are constructed according to the following principles.

Required a scale, say of 100 units to 1 inch and of 600 units in length. (*Vide* Fig. 6.) Draw eleven lines (to inclose 10 spaces) 6 inches long, parallel and equidistant (any space between these lines will do, but $\frac{1}{10}$ of an inch is usually found a convenient distance, the size of the scale being however the best guide to follow.) Divide these lines into 6 equal parts, forming 6 spaces or primary

divisions separated by vertical lines. Subdivide the lower line of the left hand primary division into 10 equal and secondary parts—from the first left hand subdivision of which draw a diagonal line to the upper intersection of the highest line with the left hand bounding extremity of the scale: from each of the other secondary divisions draw lines parallel to this and number them as in example.

The primary divisions will represent 100ths of units, the secondary divisions will represent 10ths, and the tertiary divisions each $\frac{1}{10}$ of the secondary or one single unit. (Euclid 2, VI. book.)

All diagonal scales are constructed on similar principles—but, like the plain scales, they differ in their rate of division.

On most protractors the diagonal scales bear the ratios of 200 and 400 to 1 inch.

On a scale of 100 to 1 inch, required distances of 2.5, 3.27 and 4.665. (*Vide* Fig. 6.) The dark line drawn from the second primary division 2 to the fifth secondary division on the base line will show the first distance. The dark line drawn from the point at which the seventh parallel line from the base is crossed by the second diagonal to the third primary division will give the second division.

And, in the same manner, that drawn midway between the two 6th and 7th parallels, will point the way of obtaining distances requiring 3 places of decimals.

Practice.

On the scale whose method of construction has been given of 100 to 1 inch, mark off with a red line, or with a dark pencil, spaces of 1.7, 3.45, 4.78, 5.27, 5.655 and 5.785.

Should we require to obtain these distances on the scale of 200 to 1 inch, we would either multiply the distances by 2, or obtain them in half inches and double them on the

paper—the same operation by 4 will enable us to use similarly the scale of 400 to 1 inch.

This last method has the advantage of enabling us to obtain lengths of greater magnitude than those offered by the scale.

Required a line 9.92 inches long.

Take that distance on the ½ inch scale and double it.

Required a space of 16.765 inches long.

Take very carefully that distance on the ¼ inch scale and step it four times.

One moment of ocular demonstration will show the convenience of the plotting scales to obtain equidistant or proportional spaces, &c. and in many instances to dispense advantageously with the compass.

Sector.

A sector is an instrument either of boxwood or of ivory, consisting of two moveable limbs each containing a series of lines acting in pairs, and divided so as to obtain certain dimensions according to given ratios. (Euclid 2, VI. book.)

The three principal pairs of lines contained by the sector are,

The lines marked L, or lines of lines, which are divided up to 100 equal parts and are principally used for obtaining proportionals to given lines; to divide lines into fractional parts, to construct scales, &c.

The lines marked C, or lines of chords, by means of which angles can be accurately measured to degrees and half degrees, and by which other fractional parts of degrees can be approximatively obtained. They are graduated up to 60 degrees.

The lines of Polygons marked Pol, which are used for the construction of polygons, whether in given circles or on given bases, and from 4 to 12 sides.

N.B.—It is is to be observed, that the dimensions on the sectorial lines are to be obtained from the lines, at the extremity of which there is a small brass nail.

There are other pairs of sectorial lines, of which two, marked T, are used for tangents, and are graduated up to 45° and 75°. One S for sines, graduated to 90, and one pair for secants s, and graduated to 75°.

The sector contains likewise logarithmic scales of sines, tangents and numbers.

When open to its whole length, the sector is 12 inches long, and its back is divided into 100 equal parts.

On one of its edges, the same distance is divided into 12 inches, each decimally subdivided into 10ths of inches, or in 120 equal parts to one foot.

The spaces marked longitudinally on the sectorial lengths are termed lateral distances.

The dimensions taken on the open sector, reaching to similar indices of a pair of lines having the same denomination, are termed "transverse distances."

LINE OF LINES (L).

The lines of lines are divided into 10 equal parts, termed primary divisions, each of which is subdivided into 10 secondary subdivisions, the value of each of which may optionally equal .1, 1, 10, 100, 1000, &c. Application, (*vide* fig. 7.)

1. To divide a line 2 inches long into 10 equal parts. Take the given length (2 inches) in the compass, and adapt the sector to it at the denomination required; then open the sector so that the transverse distance 10-10, equals two inches. It is evident (Euclid 2, VI. book) that the transverse distances, 9-9, 8-8, 7-7, &c. will equal .9, .8, .7 of 2 inches.

2. Show .73 of a line 2 inches long?

As in other words we require $\frac{73}{100}$ of 2 inches, open the sector so that 100-100 equals 2 inches, and adapt the compass as the radius 73-73, the distance required.

3. On a map a distance of 1-6 represents a space of 80 miles, complete the scale to 100.

The given space, 1.6 being obtained in the compass, adapt it to the transverse distance 80-80, and open the compass from 100 to 100 transverse distance, for the whole length of the scale. Divide that line into 10 equal parts, as in the first example, and subdivide decimally the first left hand primary division by repeating the same process.

4. Find $\frac{7}{9}$ of a line of any given dimension, say of 1.7 inch. Open the sector so that the transverse distance, 9-9, equals the length of the denominator (1.7), and adapt the compass at the transverse 7-7. Hence the rule applicable whether for obtaining a decimal or a vulgar fraction of a line. Open the sector so that the larger transverse distance, or space representing the length of the given line coincides with the denominator. Then adapt the compass to the transverse distance representing the numerator.

Find $\frac{72}{85}$ of .8 of a line 4 inches long.

First obtain .8 of 4 inches, and then $\frac{72}{85}$ of that distance.

Practice.

1. Find $\frac{35}{78}$ of $\frac{7}{9}$ of .87 of $\frac{2}{3}$ of a line 5.3 inches long.
2. Find $\frac{4}{7}$ of $\frac{7}{9}$ of .73 of .785 of a line 6 inches long.
3. On a map, 73 miles are represented by a space of 4.2 inches, complete the scale to 100.
4. Construct a scale of equal parts of 7 units to 1.$\frac{1}{4}$ inches.

To find a *third* proportional to two given lines, say to 4 and 6 units, compass from 0 to 6 (the second term) as a lateral distance, and adapt that space as a transverse distance from 4 to 4 (first term). Now take the transverse distance 6-6 (second term), and from 0 measure

that distance laterally. The result, 9-9, will be the third proportional.

To find a fourth proportional to three given lines, say 3, 5, and 6, take in the compass the lateral distance of the second term (5), and make it the transverse distance, 3-3 of the first term. Take the transverse distance of the third term 6-6, and measure it laterally on the scale. The result 10 will give the fourth proportional.

PRACTICE.

1. Find a third proportional to 5 and 30.
2. Find a fourth proportional to 2, 4 and 5.

LINE OF CHORDS (C).

The line of chords (*vide* Fig. 8) is divided on each limb into six parts of 10 degrees each, not equal, but bearing to each other the same ratio as those obtained by the construction of the scale of chords; each of these parts is divided into 10 parts, or single degrees, and each degree is again subdivided into two parts, or half degrees, each equal to 30 minutes.

To determine an angle by the line of chords, assume the angle to be 50°.

Draw any line AB (Fig. 9), and with any radius describe the arc BC; open the sector so that the transverse distance 60-60 may equal the radius AB, adapt the compass from 50-50, the chord of the angle required, and transfer it from B to D on the arc BC, join AD. Repeat the same operation for any other angle.

When the angle exceeds 60° divide its value by 2 or 3, as may be necessary, and after having obtained the chord of that fraction of the angle, step it twice or three times on the given arc.

As one degree 1° equals 60 minutes, and as each degree on the line of chords is subdivided into two half degrees,

it will be easy to estimate approximately any intervening number of minutes between 0 and 30′ or 30′ and 60′.

PRACTICE.

By the line of chords determine angles of 17°, 33°, 47°-15′, 56°-45′, 72°-30′, 117°-20′, 147°-40′, and 168°.

From a circle of 9′ radius cut off by the chord an arc of 42° 30′, and determine the length of the chord. (Fig. 12.)

LINES OF POLYGONS. POL. (FIG. 10.)

The line of Polygons forms the nearest scale to the two inner edges of the sector, and is marked Pol.

Let us remember that the radius of a circle is equal to the chord of $\frac{1}{6}$ of its circumference. If, therefore, we adapt any opening of the compass to the transverse distance 6-6, that distance will form the radius of a circle containing it 6 times.

Those scales have been so graduated that the transverse 5-5 will be contained 5 times in the circle of which 6-6 is the radius; the transverse 7-7 will be contained 7 times in the same circle, 8-8 eight times, 9-9 nine times, &c., and so on.

EXAMPLE. (Fig. 11.)

In a circle of .9 inches radius inscribe any polygon, say a pentagon.

With the given radius of the circle (.9) as a transverse distance, open the sector at 6-6; adapt the compass at 5-5, which will equal the side of the required inscribed pentagon.

Should we wish to inscribe in the same circle any other polygon, say a heptagon, or a nonagon, without changing the opening of the sector, the only operation would be to find the transverses 7-7 and 9-9, for the sides of the figures required.

N.B.—Should the sides of the polygons not fit accurately in the circle, the learner need not feel discouraged, as the error most likely proceeds from the thickness of the pencil, from taking his dimensions from within or without the circle, or from some other trivial cause which requires correction.

When the polygon is to be constructed on a given line or base, the operation is the converse of the preceding: adapt the sector so that the transverse distance equals the length of the given base when applied to the numbers corresponding with the denomination of the required polygon. Adapt the compass at 6-6 for the radius of the circle that will contain that polygon, with which radius, and from both the extremities of the given base as centres describe two arcs intersecting each other above it; the point thus determined will give the centre of a circle, which will contain as many chords or sides, equal to the base, as are contained in the required polygon.

Example.

On a base of .75 inches construct a regular heptagon. (Fig. 12.) Let that base equal the transverse distance 7-7, and adapt the compass at 6-6. With that distance as a radius, find the centre of the circle as just explained, and it will contain the length of the base seven times.

Practice.

In a circle of 2 inches radius construct a square, a heptagon, and a nonagon.

On bases of 1-2 inches construct a pentagon, an octagon, and a nudecagon.

From a circle of .9 inch radius cut off an arc of 42°, and determine the length of its chord, (*vide* Fig. 126).

MARQUOIS SCALES.

As we have already observed, military students are usually provided with a most useful pair of scales, termed "Marquois," from the name of the inventor. A proper acquaintance with these scales will not only greatly facilitate the performance of works of detail, but whilst in a great measure obviating the necessity of using the compass, will produce those constructions with much greater rapidity, neatness, and accuracy. The proof of their construction depends on Euclid 2, VI. book.

A set of Marquois scales consists of two rulers and a triangle. The rulers are each 12 inches long, and so graduated as to contain on each of their edges one pair of scales—the one nearest the edge being termed the "artificial scale," and that immediately below it the "natural, or true scale;" both these scales are divided into spaces of 10 units, the only difference between them being that the space of 10 units on the artificial scale is ever equal to three spaces of 10 units on the natural, making therefore 3 units of the natural equal to one of the artificial.

In the centre of these scales, which are eight in number, and opposite the zero is an index marked either 20, 25, 30, 35, 40, 45, 50, and 60, and denoting the ratios into which the space of one inch is divided on each of the respective scales.

The triangle which accompanies these scales has its two shortest sides meeting at right angles, in such a manner that the hypothenuse is exactly three times as long as the shortest side, or perpendicular. The hypothenuse of the triangle bearing, therefore, the same proportion to the perpendicular as the artificial scale bears to the natural.

To obtain measurements by these scales, (*vide* Fig. 13) say to draw two parallel lines at $\frac{16}{40}$ distance from each other.

Place the base AB of the triangle so as to coincide with

the first given line GL, from which the distance is to be obtained; place the ruler (with, in this case, index 40) diagonally, so that its edge coincides with the hypothenuse AC of the triangle, the arrow in its centre coinciding also with the zero O on the scale; then, whilst keeping the ruler steady with the left hand, slide the triangle with the right till the arrow is transferred and coincides with whatever may be the measurement required on the artificial scale—in this case $\frac{16}{40}$. The perpendicular B *b* will be found equal to one third of the space passed over by the arrow of the hypothenuse, and equal to that which would be obtained with the compass on the natural scale.

The following practice is recommended to those who wish to become familiar with the use of these scales:—

Draw two lines seven inches in length parallel and one inch apart; divide the space between these lines into 14 compartments each $\frac{1}{2}$ an inch in breadth (*vide* Fig. 14), and subdivide by Marquois each of these compartments of 1 inch high by .5 inch broad into given numbers of equal parts, as already given in the explanation of plain scales.

The scale of 55 being wanting, the division of the inch into 11 equal parts is omitted, as well as that of 13, 17, and 19, which would be difficult and exposed to inaccuracy.

Dimensions can be obtained on the natural scales by means of the compass, precisely in the same manner as on the plain decimal scales affixed to the protractor, from which they do not differ in principle, but only in length.

Should we require to obtain through these scales the length of a line decimally given, say 3.5 inches, it is evident that that distance multiplied by the index of any scale will change its denomination, therefore let us assume the scale of 20, then, $3.5 \times 20 = \frac{70}{20}$, the distance required.

Take 5.83 by the scale of 30, then $5.83 \times 30 = \frac{174 \cdot 9}{30}$.

Take 2.9 by 45, then $2.9 \times 45 = \frac{130 \cdot 5}{45}$, &c.

In practice, the student will often discover some new means of using these scales with advantage, especially in the construction of diagonal and other scales, and in their application as substitutes to the triangle and T square.

THE PROPOSITIONS OF MOST FREQUENT APPLICATION.

Those on which depend the construction of most Geometrical diagrams, the solution of most questions likely to be asked, are chiefly founded on the following Theorems:

Euclid, Book I.

Prop. 15.—*Theorem.* If two straight lines cut one another, the vertical or opposite angles shall be equal.

Prop. 29.—*Theorem.* If a straight line fall upon two parallel straight lines, it makes the alternate angles equal to one another; and the exterior angle equal to the interior and opposite upon the same side; and likewise the two interior angles upon the same side together equal to two right angles.

Prop. 32.—*Theorem.* If one side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of every triangle are equal to two right angles.

Prop. 34.—*Theorem.* The opposite sides and angles of parallelograms are equal to one another, and the diameter bisects them, *i.e.*, divides them into two equal parts.

Prop. 35.—*Theorem.* Parallelograms upon the same base and between the same parallels are equal to one another.

Prop. 37.—*Theorem.* Triangles upon the same base and between the same parallels are equal to one another.

Prop. 47.—*Theorem.* In any right angled triangle, the square which is described upon the side subtending

the right angle, is equal to the squares described upon the sides which contain the right angle.

Euclid, Book III.

Prop. 3.—*Theorem.* If a straight line drawn through the centre of a circle bisect a straight line in it which does not pass through the centre, it shall cut it at right angles, and if it cuts it at right angles it shall bisect it.

Prop. 31.—*Theorem.* In a circle, the angle in a semicircle is a right angle, but the angle in a segment greater than a semicircle is less than a right angle, and the angle in a segment less than a semicircle is greater than a right angle.

Euclid, Book VI.

Prop. 2.—*Theorem.* If a straight line be drawn parallel to one of the sides of a triangle, it shall cut the other sides or those produced proportionally, and if the side or the sides produced be cut proportionally, the straight line which joins the points of section shall be parallel to the remaining side of the triangle.

Prop. 33.—*Theorem.* In equal circles, angles whether at the centres or circumferences have the same ratio which the circumferences on which they stand have to one another, and so have the sectors.

Many of the following problems can be worked in several ways, but to avoid confusion, I will as much as possible confine myself to that single solution which I consider the best and most simple, indicating the propositions of Euclid demonstrating them.

All the problems and illustrations given in this work should be constructed at least twice or three times larger than those exemplified in the diagrams.

PRACTICAL GEOMETRY.

1. At a given point C upon a given line AB to erect a perpendicular. (*Euclid* 4 *and* 8, I.)

From C as a centre and with any radius describe the semicircle DE. From the points D and E as centres, and with any radius greater than DC or EC, describe arcs intersecting each other in F, join CF, the perpendicular required.

2. At the extremity B, of a given line AB, to erect a perpendicular. (*Euc.* 31, III.)

Any where above AB determine the point C.

From C as a centre, with radius CB, describe the arc DB, intersecting AB in E. Through C draw the line ECD, and join DB, the perpendicular required.

Another method, exemplified at the extremity A of the given line, is also often useful.

From A as a centre, and with any radius, describe the arc FG. From F, with radius FA, describe the arc AH; and from H, with the same radius, describe the arc AK. From H and K, as centres, describe two arcs intersecting in L, and draw LA, the perpendicular required.

3. From a point in space C, nearly opposite the extremity of a given line AB, to drop a perpendicular. (*Euc.* 31, III.)

Draw any line CD, cutting AB in D, and bisect it in E. From E as a centre, with radius EC, describe the arc CBD, and draw the perpendicular C B.

4. From a point in space G, over a given line AB, to drop a perpendicular. (*Euc.* 4 *and* 8, I.)

From G as a centre, with any radius greater than its distance from the given line, describe an arc cutting AB in H and K.

From H and K as centres, describe two arcs intersecting in L, join GL, meeting AB in M.

5. Through a given point C, to draw a line parallel to a given line AB. (*Euc.* 27, I. *and* 27, III.)

From C as a centre, with any radius, describe the arc DE.

From D as a centre, with the same radius, describe the arc CG.

Make the chord DE equal to the chord CG, and draw the line CE, which will be the parallel required.

6. To bisect a given line AB. (*Euc.* 4 *and* 8, I.)

From the extremities of the line A and B, and with any radius greater than half the whole length of the line, describe arcs intersecting above and below it, in C and D.

The line CD being drawn, will bisect AB.

7. To divide a given line AB into any number of equal parts (say nine.) (*Euc.* 2, VI.)

From A draw the line AC of any convenient length, and at any angle with AB (an angle from 45° to 60° is usually found the most convenient) with any convenient distance on the compass step on AC nine equal parts.

From the ninth, or last space C, draw a line joining CB, and from each of the other points draw parallels to CB.

N.B.—It is evident that this problem, which is of most frequent occurrence in Geometrical drawing, useful in the construction of scales, and on which the construction of the sectoral lines depends, will also enable to divide given lines according to any given ratio.

8. To bisect a given angle, CAB. (*Euc.* 8, I.)

From A as a centre, with any radius, describe an arc cutting AB and AC in D and E. From D and E as centres, describe arcs intersecting in G, draw AG the bisecting line required.

9. Upon a given line DE to construct an angle equal to the given angle, BAC. (*Euc.* 8 I. *and* 27, III.)

From A as a centre, with any radius, describe the arc HG intersecting AB and AC.

From D as a centre, with the same radius, describe the arc KL.

Measure with the compass the chord GH, and transfer it to KL, join DL. The angle KDL will be equal to BAC.

10. Upon a given line HK, to construct any rectilineal figure similar and equi-angular to a given figure of the same, or of a different size. (*Euc.* 8, I. *and* 27, III.)

Let AB, CD, EF, and G, be the given figure, and HK the given line.

From A as a centre, draw the lines AC, AD, AE, AF, AG, and from B draw similarly the lines BG, BF, BE, BD and BC. From A and B as centres, describe the arcs 1-6 and 7-12. From H and K as centres, describe with the same radius the arcs 13-18, and 19-24. From the point 1 measure separately all the chords 1-6, 1-5, 1-4, 1-3, 1-2, and transfer them to 13-18, 13-17, 13-16, 13-15, and 13-14; repeat the same operation with the chords 7-12, 7-11, 7-10, 7-9, 7-8, and transfer them likewise to 19-24, 19-23, 19-22, 19-21, and 19-20. From H and K respectively draw indefinite lines through 14, 15, 16, 17 and 18, and intersect them by lines drawn through 20, 21, 22, 23, and 24.

Through these points of intersection draw the lines HL,

LM, MN, NO, OP, and PK, which will form on the base HK a figure similar and equiangular to the given one.

11. To determine the direction of a line that would pass from any intermediate point through an angle formed by the direction of two lines, which, if produced, would fall beyond the limits of the drawing.

Let AB and CD be the given lines, and E the intermediate point, through E draw BED, and draw AC parallel to it. Draw the diagonal AD, and draw EF parallel to CD, and FG parallel to AB. Join EG, the line required.

12. To determine the direction of a line that would bisect a given angle formed by the direction of two lines, which, if produced, would fall beyond the limits of the drawing, and to inscribe a circle of a given radius tangent to both lines.

• Let AB and CD be the given lines, 3 inches long, so placed that A is 1 inch from C, and B 2 inches from D. Let the diameter of the proposed circle equal 1·5 inches.

Draw anywhere EF parallel to CD, so as to form the angle FEB. From E as a centre, with any radius, describe the arc GH. Draw the chord GH produced to K, and bisect GK (prob. 6) in LM.

Draw AN perpendicular to AB, and equal to ·75 inch, the radius of the proposed circle. Draw NO parallel to AB, and cutting LM in P.

From P as a centre, with the length AN as a radius, describe the required circle, which will be tangent to AB and CD.

13. Through three given points, A B and C, not in the same straight line, to describe a circle. (*Euc.* 39, III.)

Bisect the distances AB and BC (prob. 6) by straight lines meeting in D.

From D as a centre, with radius DA, describe the circle which will pass through the three points. N.B.—Problems to find the centre of a circle, to complete the circle whose centre is lost, to describe a circle round a triangle, &c., may be solved in a similar way.

14. From a point A, without the circumference, to draw a tangent to a circle, BCD. (*Euc.* 16 *and* 31, III.)

Join the centre E to the given point A, and bisect AE in G.

From G as a centre, describe the arc EHA, cutting the circumference in H. Join AH the tangent required.

15. At a given point A, on a given circumference ABC, to draw a tangent. (*Euc.* 4 *and* 8, I.)

From the centre D draw DAG, and make AG equal to AD, bisect DG, and the bisecting line HK will be tangent at A to the given circumference.

16. Through two points on the circumference of a circle 90° apart, to draw tangents. (*Euc.* 16, III.)

Let ACG be the given circle, A one of the given points, and B the centre of the circle. From A as centre with radius AB, describe the arc BCD. Make CD equal to CB, and with the same radius bisect CD by arcs intersecting in E, draw EA, the first tangent cutting CD in H. From H, with the same radius, cut the circumference in K. Join HK, the second tangent required, which will be at right angles with AH.

17. The circle and tangent being given, to find the point of contact. Let the line AB be tangent to the circle CD. (*Euc.* 4 *and* 8, I.)

From centre E, with any radius greater than that of the given circle, draw arcs cutting AB in G and H. From G and H respectively, describe arcs intersecting above AB in K. Join KE, cutting AB in L, the required point of contact.

18. On a given straight line DE, to describe a segment of a circle that will contain an angle equal to a given angle ABC. (*Euc.* 33, III.)

Make the angle EDF equal to the angle ABC and draw DH at right angles to DF. Bisect DE in G and erect the perpendicular GK cutting DH in L. From L as a centre with radius LD describe the segment DME, every angle contained by which will be equal to the given angle ABC.

N.B.—The student will find this proposition most important, and of a very frequent use in the questions given at the examinations.

19. From a given circle, to cut off a segment that shall contain an angle equal to a given angle. (*Euc.* 34, III.)

Let ABC be the given circle, DEG the given angle. (*This proposition is the converse of the preceding one.*)

From the centre L draw LA as radius, and make AH perpendicular to it and tangent to the circle: make the angle MAH equal to the angle DEG: all the angles contained by the segment AMBC will be equal to the angle DEG.

20. On a given straight line AB to describe an isosceles triangle having a given vertical angle EFG.

Produce AB in C and make the angle CBD equal to the angle GFE. Bisect the angle ABD in H and draw

BH: make the angle BAH equal to the angle ABH and the angle AHB will be equal to the angle EFG.

21. To construct an equilateral triangle of a given height AB.

Make the line AB equal to the given height, through A draw CD perpendicular to it and through B draw EF parallel to CD. From B as a centre with any radius describe the semicircle EGF and make the chords EK and FH equal to the radius BE or BF. From B through K and H respectively draw the lines BK and BH produced so as to meet CD: BCD will be the triangle required.

22. To construct a triangle whose sides equal 3 given lines A-B, B-C and C-A

A ———————————— B
B ——————————— C
C —————————— A

Draw A′ B′ equal to AB.

From A′ as a centre with radius AC describe an arc, so that

From B, as a centre with radius BA it may be cut by another arc in C.

ABC will give the triangle required.

23. To reduce any rectilineal figure to a triangle, or to any other irregular polygon of a less number of sides. (*Euc.* 37, I.)

Let ABCDEF be the given figure.

Produce indefinitely on both sides the base AF towards K and G.

Join AC and draw BG parallel to AC: join CG.

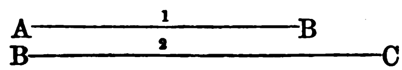
Join DF and draw EH parallel to DF and join DH.

The figure GCDH will equal the given figure: now to

reduce it to a triangle, join CH and draw DK parallel to CH and join CK.

The triangle GCK will equal in area the figure ABCDEF.

24. To find a third proportional to two given lines AB and BC. (*Euc.* 11, VI.)

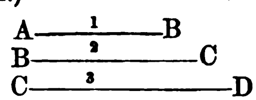


Draw any angle DEG and make EH equal to AB, and EK equal to BC, and join HK.

Make KL equal to AB and draw LN parallel to KH.

HN will be the third proportional required.

25. To find a fourth proportional to three given lines. (*Euc.* 11, VI.)



Draw two lines forming any angle DEG.

And make EH equal to AB, EK equal to BC and join HK.

Make HL equal to CD and draw LM parallel to HK.

KM will be the fourth proportional required.

26. To find a mean proportional between two given lines. (*Euc.* 6 *and* 8, VI.)

Let A——B B—C equal the given extremes.

Draw A—C equal to the sum of AB+BC and bisect it in D.

From D as a centre describe the semicircle AEC and draw BE perpendicular to AC: BE will be the mean proportional required.

27. To construct an equilateral triangle equal to a scalene ABC.

On AB as a base construct the equilateral triangle ABD and parallel to AB, draw CE meeting DA produced: find the mean proportional AF between DA and AE, and from A as a centre, describe the sector FG whose side AG will form the base of the equilateral triangle required.

28. To transform any irregular figure into a regular polygon of equal area, say, the irregular heptagon ABCMDEF into a pentagon.

Reduce the irregular heptagon to the triangle KLM, (*problem* 23).

Divide the base KL into as many equal parts as there are sides in the polygon required (in this case, five). From K as a centre, with radius KN ($\frac{1}{5}$ of KL), describe the arc NO on MK produced: draw KP mean proportional between MK and KO and from K as centre, with radius KP describe a circle which will inscribe the polygon required, and of the same area as the given figure.

N.B.—The inscribed polygon may be obtained either Geometrically, by sector, or by determining the angle MKR (in this case 72°) equal to the value of the angle at the centre of the pentagon.

29. To transform a given irregular figure to a square of equal area.

Reduce the figure to a triangle ABC. (*Euc.* 37, I.)

From the apex B draw BD perpendicular to AC, and bisect BD in E, through E draw GH parallel to AC, and draw GA and CH parallel to BD. Find the mean proportional AK between the two sides of the parallelogram AC and AG: AK will be the side of the square required.

30. To construct a figure of given area, say a triangle of 3.73 square inches.

Make a parallelogram of 1 inch side by 3.73 inches base; whose area will evidently equal 3.73 square inches. Reduce it to a triangle by doubling the height of the side and drawing a diagonal from the apex to the further extremity of the base.

N.B.—We have already explained how any triangle could be transformed into any polygon of equal area. Scale $\frac{1}{4}$.

31. To divide a straight line into mean and extreme ratios.

Let AB be the given line.

Draw AC perpendicular to it, and equal to half its length.

Join BC: With radius CA describe the arc AE and with radius BE describe the arc ED, then BD will be the mean and DA the extreme ratios.

BA : BD : : BD : DA.

32. To transform a given triangle into one similar, and equiangular to another triangle of a different shape and having a different area.

Let ABC be the triangle to be reduced to the shape of the triangle DEF :

Make the angle BAG equal to the angle EDF.

Draw CG parallel to AB and join GB :

Make the angle ABK equal to the angle DEF.

Then the triangle ABK will be similar to the triangle DEF, but greater than the triangle ABG by the triangle GBK.

Draw GL parallel to BK.

The triangle ALG will be similar to DEF, but of less area than ABG by the contents of the triangle LBG.

Find GM mean proportional between AG and GK, and from A as a centre, with radius AM, describe the arc MN, and draw NO parallel to BK.

The triangle ANO will be similar and equiangular to the triangle DEF, and equal in area to the triangle ABC.

33. To reduce the triangle ABC to a polygon of equal area but similar and equiangular to another polygon, DEFGH.

Make the triangle ABL similar and equiangular to the triangle FGH, and draw CK parallel to BA.

Reduce to a triangle GHI, the polygon DEFGH.

Divide BK at M, in the same ratio as IG is divided in F.

Find BO mean proportional between BL and BM, and draw OP parallel to AL.

On OP as a base, construct the trapezium OPQR, similar and equiangular to the trapezium HFED, and the polygon BPQRO will be equal in area to the triangle ABC and similar and equiangular to the irregular polygon DEFGH.

34. To construct a square equal in area to the sum of any number of squares (say 3) whose sides are given. Let the sides of the squares equal 1, 1.25 and 2 inches respectively. (*Euc.* 47, I.)

Draw AB equal to 1 and AC equal to 1.25 inches, and at right angles to AB: Join CB. Draw BD equal to 2 inches, at right angles to CB, and join DB. CD will be the side of the square equal in area to the sum of the three given squares.

35. Construct a circle equal in area to the sum of any given number of circles (say 3) whose radii equal 1, 1.25 and 1.5 inches respectively.

Make AB equal to the radius 1, and draw AC at right angles to it, equal to the radius 2, join CB, draw BD at right angles to CB and equal to the radius 3, and join CD; from D with radius DC describe a circle whose area will be equal to the sum of the three given circles.

36. To divide a circle into any number of concentric rings whose areas will bear given ratios to each other.

N.B.—This proposition is the converse of the preceding.

Let A be the centre and BCDE the circumference of the proposed circle; draw the radius AB and divide it according to the required ratios, (say 3 equal parts) A—1, 1—2, 2—B. Bisect AB in F, and from F as a centre describe the semicircle BGA. On AB erect the perpendiculars 1—3 and 2—4.

A circle described from A, with radius A—4, will cut off a ring equal in area to ⅓ of the area of the circle.

A second circle described from A, with radius A—3, will give a second ring equal in area to the first and to the remaining central circle.

37. On a given base, AB, to construct a regular pentagon.

Bisect AB in C, and erect the perpendicular CF.

Make CD equal to AB, and AD produced towards E.

Make DE equal to AC, and with the radius AE describe the arc EF; from A, B and F as centres, with radius equal to AB, determine the points GH, and complete the pentagon.

38. On a given line, AB, to construct regular polygons from the hexagon or the dodecagon, and subsequently from that of 12 to 24 sides.

Bisect AB and draw the perpendicular CF indefinitely

produced towards D. From A and B as centres, and with the radius AB, describe the two indefinite arcs intersecting in F. Divide the arc BF in six equal parts, and from F as a centre describe a series of concentric circles, 1—7, 2—8, 3—9, 4—10, 5—11, 6—12, meeting on CFD. The circle drawn from centre F, with radius EB, will contain AB 6 times. The circle described from centre 7, with radius 7B, will contain AB 7 times. The circle described from centre 8, with radius 8B, will contain AB 8 times, and so on till the circle drawn from centre 12, with radius 12B, which will contain AB 12 times.

For the second part of the problem divide the arc AF into 12 equal parts, and from F take as many parts as are required above twelve to complete the required polygon.

Should we, for instance, require a septodecagon, or polygon of seventeen sides: from F we should take FG $= \frac{5}{12}$ of the arc FA, and transfer it on the perpendicular D in H, then make HK equal to HA, the point K would be the centre, and KA the radius of a circle containing 17 times the base AB.

39. A circle being given, to find the length of the side of any polygon that it might contain, from the triangle to the dodecagon.

Draw the circle BCDE and its two diameters BD and CE, intersecting at right angles in A. From D, with radius DA, describe an arc cutting the circumference in F and G; join FG, whose length will equal that of the side of an equilateral triangle contained by the circle.

The length BC will give the side of a square.

From H, centre of FG, with radius HE, describe the arc EK the distance EK will equal the length of the side of a pentagon.

FD equals the side of a hexagon, HG the side of a heptagon.

BL, the chord of half the arc BC, equals the side of an octagon.

From B, with radius BF, describe the arc FM, and from M transfer the distance BM to N; the line NC will equal the side of a nonagon.

AK will be the side of a decagon. From D, with radius DH, describe the arc HO, and from H, with same radius, cut it in P; then from O as centre, with radius OP, describe the arc PR, and join HR for the side of an undecagon.

The distance EF will equal the side of a dodecagon.

40. The transverse and conjugate axes of an ellipse being given, to find the foci, and to draw the figure by means of loci determined by radii vectors.

Let AB equal the transverse or major axis, and CD intersecting AB at right angles in E, equal the length of the conjugate or minor axis; then, from C and D respectively as centres, with radius equal to AE, determine the points GG, the foci of the ellipse.

On one of the semi-major axes, either AE or EB, take any point F, and with the distance FB as radius, and the points GG as centres, describe the arcs KKKK; with distance FA as radius, and the points GG as centres, describe arcs intersecting KKKK in LLLL (those points thus determined are termed the loci, and the spaces used as radii to determine them are termed radii vectors).

Take any other point, H, or as many more as may be required, and repeat the same operation. A curve drawn by the hand, and passing through every locus, will complete the ellipse.

This is the true mathematical ellipse, but among the

many different modes of constructing that figure, the most simple and practical is perhaps the following:—

41. The length of the two axes being given, to construct the ellipse by means of a slip of paper.

On a slip of paper about .1 of an inch broad, determine the distances GK, equal to half the major axis AE, and HK, equal to half the minor axis DE. Let AB and CD, intersecting in E, as in the previous example, represent the transverse and conjugate axes.

Move the slip of paper in such a manner that the points H and G shall always coincide with the two axes, *i.e.*, the point G with some part of the minor axis CD, and the point H with some part of the major axis AB, determining at the extremity K of the slip, with a fine pencil, as many points or loci as may be desired, as 1, 2, 3, 4, 5, 6, &c.; through those points determine the circumference of the ellipse.

42. At a given point on its circumference to draw a tangent to an ellipse.

Let A be the given point and B and C the foci.

Join BA and CA, and bisect the angle BAC by the perpendicular DE.

Through A draw EG at right angles to DE: EG is the tangent required.

43. From a point in space K to draw a tangent to an ellipse.

Draw any three lines, KL, KM, KN, each one cutting the ellipse in two points, OL, PM, and QN; join MO and LP, also MQ and NP, by lines intersecting; through the intersections TV draw the line RS, which will determine the points of contact of the tangents KS and KR.

44. An ordinate AB, and the abcissa CD of a parabola being given, to describe the curve by means of intersecting points.

Make AB equal to the given ordinate, and bisect it in C.

Draw CD perpendicular to AB, and equal to the height of the abcissa.

Bisect AC in E, and produce CD towards H and F.

Draw EF at right angles to DE.

From D as a centre, with radius CF, determine the points G for the focus and H for the directrix. Between G and C draw any number of ordinates parallel to AB, and from G as a common centre, with the distance

1-H	as a radius, determine the points	KK	intersecting the ordinate	1
2-H	,,	LL	,,	2
3-H	,,	MM	,,	3
4-H	,,	NN	,,	4

through these points from the vertex D draw the curve by the hand.

45. At a given point D on its curve, to draw a tangent to a parabola.

Let ABC be the parabola, EB its abcissa indefinitely produced towards F, and D the given point.

From D draw the semi-ordinate DG, at right angles to EB.

From B as a centre make BH equal to BG, and through D draw HK, the tangent required.

46. The transverse and conjugate axes of an hyperbola being given, to construct the curve.

Make the line AB equal to the length of the given conjugate axis, bisect it in C, and produce it indefinitely on both sides towards X and Z.

At right angles to AB draw DAE equal to the length of the transverse axis.

From C as a centre draw DCF and ECG for the asymptotes.

From C, with radius CD, describe the circle DHEFKG, determining the two foci H and K; on the abcissa AX take any point L, and with distance LA as a radius, and from the focus H as a centre, describe the arcs NN.

With LB as radius and from K as a centre, cut those arcs in OO.

Take any other point, M, and with distance MA from centre H, determine the arcs PP. With distance MB as radius, and from centre K, cut those arcs in QQ.

And so on with any other number of points, the arcs produced by which will approach nearer and nearer to the asymptotes, but never touch them; through these points the curve is drawn by the hand.

47. To describe a circle equal in area to a given ellipse, whose transverse AB, and conjugate diameter CD intersect in E.

Find EG, mean proportional, between AE and EC. From centre E, with radius EG, describe the circle GHIK, whose area will equal that of the ellipse.

48. On a given major axis AB, to construct an ellipse equal in area to the given circle CDEF, whose centre is made to coincide with the centre of the given line.

Draw the diameter EC of the circle perpendicular to AB, join AC, and bisect it in G. Draw GK perpendicular to AC, and from K as a centre, with radius KC, describe the arc CL. From M as a centre, with radius ML, describe the arc LO, O′. AB will be the major, and O, O′ the

minor axis of the ellipse, whose area (when completed in the usual way) will equal that of the circle CDEF.

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49. The following practical approximations to mathematical impossibilities may also at times prove useful. They are founded on the assumption that every *minute* space taken on the circumference of a circle is equal to a straight line.

To construct a triangle equal in area to a given circle.

Let ABCD be the given circle, whose diameter AC BD intersect at right angles in M. Draw AE AE′ tangent to the circle. With a very minute opening, step the compass on the arc from A to B (each of these minute spaces will be considered as bases of equal isosceles triangles, having their common vertex at the centre of the circle M.) The same number of those equal spaces being transferred on the tangent AE from A to G, and the line MG being drawn will evidently present a right angled triangle equal in area to the quadrant AMB; on AE′ make AG′ equal to AG. The triangle G′ MG will equal the semicircle DAB. If now we join CG, CG′, the triangle G′C, G will evidently equal the area of the circle.

We have already explained how a triangle might be converted into a parallelogram, a square, or into any other polygon.

50. To reduce to a circle the scalene triangle ABC.

Bisect the base AB in D, and erect DF perpendicular to it. Draw CF parallel to AB, from F as a centre describe the circle DOP, and reduce this circle to the triangle FGH: find DI mean proportional between DA and DG, and draw IK parallel to GF. From the point K as a centre, draw the circle DMN, which will be equal in area to the triangle ABC.

The following exercises, as well as those throughout the work, are founded on questions given at the Artillery, and other examinations.

EXERCISES.

1. Draw a straight line 2 inches long, intersecting at right angles another straight line 3 inches long.

2. Draw a straight line 2.78 inches long, and from its right extremity drop a perpendicular without adding to the length of the line.

3. At the left extremity of the same line, and without producing it, erect a perpendicular.

4. Determine (with compass only) a point in the prolongation of a straight line, which is not to be produced, at 1.7 inches distance from it.

5. Divide a line 3.7 inches long into eleven equal parts (by construction.)

6. (With compass) through a point at 1.685 inches above the right hand extremity of a line 4 inches long, draw a line parallel to it, and of equal length.

7. Describe a circle of 1.64 inches radius, and draw two tangents intersecting each other from two points on the circumference 90° apart, erect perpendiculars to both tangents at points one inch distant from their intersection.

8. Draw three circles, each with a diameter of 1.5 inches, so that each circle may touch the other two, ink in one circle with a thick line, one with a thin line, and the third with a dotted line.

9. Draw three circles, whose diameters are respectively 1.7, 1.9 and 2.6 inches, so that each circle touches the other two.

10. Draw two straight lines, 1.85 inches long, forming an angle of 105°, and draw a circle of .69 of an inch radius, touching both lines.

11. Assume two points A and B, 1.68 inches apart, and with a radius of 1.88 inches describe an arc passing through them.

12. Assume 3 points, A, B, and C, not on the same straight line, and describe a circle passing through them.

13. Draw two lines, AB, BC, 3 inches long, forming at B an angle of 69°, and bisect the angle.

14. Draw two straight lines, AB and CD, each 6.5 inches long, and so inclined towards each other that A is 1.2 inches from C and B is 2·78 inches from D. Draw a straight line, which, if produced, would pass through the intersectiou of their prolongations.

15. Describe a circle of .78 of an inch radius, which shall be tangent to two lines, forming an angle of 47°.

16. Describe a circle of 1.7 inches radius, and from a point at 1.9 inches from its centre, draw a tangent to it, and on the opposite side, at a given point on the circumference, draw a tangent without using the centre.

17. Draw two lines, AB and BC, each 5 inches long, and forming an angle of 30°, describe a series of circles touching each other and tangent to the lines, commencing with a radius of .4 of an inch.

18. Draw tangents to two given circumferences, whose centres are, at 4 inches distance from each other, and whose radii are 1 and 1.7 inches respectively.

19. Draw nine parallel lines half an inch apart, passing through points .8 of an inch apart, on a line 7.2 inches long.

20. Find the mean proportional between two given lines 2.62, and 3.46 inches long.

21. Find the third proportional to two lines respectively 2 and 2.9 inches long.

22. Find a fourth proportional to three lines respectively 2, 3.2 and 4 inches long.

23. Cut a straight line 5.7 inches long into extreme and mean ratios.

24. (By compass only) with a radius of 4 inches, describe an arc of 120°, and determine angles of 15°, 30°, 45°, 60°, 75, and 90°.

25. Draw arcs of 68°, 105°, and 169° respectively, tangential to each other.

26. Construct a triangle, having two of its sides 3 and 2.85 inches respectively, and the included angle 58°, figure the remaining angle and the length of the third side.

27. Construct a triangle equal to the above, and in it inscribe a circle.

28. Assume three points A, B and C, so that the lines joining AB and BC are respectively $1\frac{5}{8}$ and 2.1 inches, and make with each other an angle of 132°, describe a circle passing through the three points.

29. Construct a triangle ABC, of which the base AB equals 2.72 inches. The angle BAC 58° 45′, and the angle ABC 46° 15′, find the length of the sides AC and BC with minute accuracy, and ink in the triangle with a very fine line.

30. Reduce a square of 1.9 inches side to an isosceles triangle of equal area.

31. Draw any six sided figure of no less area than 4 square inches, and reduce it to a triangle of equal area.

32. Construct a triangle having its sides AB = 1.7, BC = 2.85, and CA = 3.16 inches respectively, and reduce it to a right angle triangle of the same altitude.

33. A triangle has two of its sides equal to 180 and 260 yards, and the included angle is 68°, give the value of the remaining angles and side and reduce the figure to a

rectangular parallelogram of the same area. Scale 60 to 1 inch.

34. A triangle ABC has its sides respectively 1.4, 2.1, and 2.4 inches long, determine by *scale of chords* the number of degrees and minutes contained by each angle; reduce the figure to a square.

35. From a point AB in a line C, make an angle, ABD, equal to 105°, using compass only, without protractor or sector.

36. The sides of a triangular piece of ground are respectively 360, 640, and 730 yards; draw its plan. Scale 200 to 1 inch.

37. Construct an equilateral triangle 2.5 inches high, circumscribe it by a circle, and in it inscribe a square.

38. Around a square construct a triangle having two of its angles respectively 57° and 69°, and in the square inscribe an equilateral triangle.

39. Describe a circle of 1.3 inches radius, and from a point at 2.4 inches from its centre draw a tangent to it. On the opposite side draw a tangent through a given point on the circumference, without using the centre.

40. From a circle 1.7 inches radius cut off two segments containing angles of 46° and 98°.

41. On a base of 3.12 inches describe a segment of a circle contining an angle of 123°.

42. Construct a triangle having two of its angles equal to 42° 45′ and 107° 15′ respectively, and the side opposite the larger angle 3.7 inches long; find the length of the other sides and figure them.

43. Describe a circle with a radius of 2.25 inches, and draw a chord cutting off from it an angle of 47°.

44. Draw a straight line 3.48 inches long, and divide it

in the proportion of the numbers 8, 2, 5, and 3; figure the spaces.

45. In a triangle, ABC, AB is 160 yards, BC 190 yards, and AC 280 yards; find by construction a point X, when the angle AXB = 61.30, and BXC = 43.45°. Scale 100 to 1 inch.

46. From a point, A, the angles between the points B and C and C and D are 42° and 57°; the lines joining BC and CD are respectively 900 and 1600 yards long, and form at C on the line nearest A an angle of 140°. Find the point A and determine the distances of the lines AB, AC, and AD. Scale 500 to 1 inch.

47. The distance from A to B = 2.7 miles, and from B to C 1.85 miles, and these three stations are in the same straight line. I travel (not in the same straight line) from C to the station D, and then observe that the stations B and C subtend to my eye an angle of 47°, and the stations A and B an angle of 32°; find the distances of the station D from A, B and C.

48. Construct a square of one square inch area, and a parallelogram of 1 inch wide, whose area equals 3.72 square inches.

49. Construct a triangle of 5.58 square inches area, one of whose angles = 48° 45′, and another angle 56° 30′.

50. In a triangle, ABC, the line AB = 277 yards, the opposite angle C = 83°, and the side CB = 188 yards; construct the figure, and from it subtract a triangle equal to $\frac{1}{5}$ of its area, reducing the remainder to its original shape.

51. Step 12 equal distances of .3 inch along a line, from every alternate point of division of which as a centre, describe a semicircle of .3 inch radius, to be alternately on opposite sides of the line.

52. The three sides of a triangle are respectively 3, 4.5, and 5 inches long; reduce the figure to a regular pentagon of equal area.

53. Construct the figure denoted by the following measurements:—

AB = 275 yards.

BC = 180 yards.

AC = 350 yards, the angle BCD = 147°.

CD = 185.

DE = 265.

BE = 325.

Find the distance from A to E and note it in yards. Reduce the figure to a triangle of equal area, and note its contents in square yards. Scale 100 to 1 inch.

54. Find and figure the length of the mean proportional between two lines respectively 2.76 and 1.3 inches long.

55. A four sided field has its sides AB = 188 yards, BC 92 yards, CD 117 yards, AD 78 yards, and the distance from D to B = 132 yards. Construct the figure and show the distance between the points A and C. Scale 50 to 1 inch.

56. Reduce the above to a square, showing its contents in square yards.

57. Construct on a scale of 50 yards to 1 inch the irregular polygon of which the following are the dimensions:

$\left.\begin{array}{l}\text{AB}=136 \text{ yards} \\ \text{BC}=227 \quad ,,\end{array}\right\}$ AC = 205 yards.

CD = 135 ,, AD = 186 yards.

The angle BAE = 142, and the angle CED = 118°.

Round off each of the angles of the figure with arcs of circles of .75 inch radius.

58. Construct a six sided figure, ABCDEF, and reduce it to a triangle of equal area. Scale 400 to 1 inch.

$\left.\begin{array}{l}AB=600\\BC=800\end{array}\right\}$ the angle ABC=150°.

CD=700 the angle BCD=120°.

AF=900.

FE=850.

DE=550.

59. On a scale of 40 to 1 inch construct the irregular figure, ABCDEFG, according to the following dimensions:

$\left.\begin{array}{l}AB=164 \text{ feet}\\BC=117 \quad ,,\end{array}\right\}$ the angle ABC=143°.

CD= 96 ,, the distance from B to D = 172 feet.

AG=110 ,, and from B to G = 218 feet.

GF= 84 ,, the angle AGF = 113°.

FE= 65 ,,

DE=127 ,,

60. Draw twelve concentric rings .17 of an inch apart, the outer one having a radius of 3.2 inches, lay a flat wash of indian ink between every alternate ring.

61. Draw three concentric circles of 1.7, 2.5, and 3.2 inches diameter, divide the circumference of the outer circle into 15 equal parts, and from these points of division draw lines radiating towards the centre, and stopping alternately at the middle and inner circles.

62. Divide a line .92 inches long into 14 equal parts.

63. By the diagonal method, divide a line 1.32 inches long into 140 equal parts.

64. Construct a square equal in area to the sum of four squares of 1.3, 1.5, and 2.1 inches side, and of a circle of 2 inches diameter.

65. Describe a parallelogram of 4.27 inches area, whose height equals 1.5 inches.

66. Construct a square of 4.58 inches area.

67. Construct an isosceles triangle having a vertical angle of 68°, equal in area to the sum of 5 squares, whose sides are respectively $\frac{1}{3}$, $\frac{2}{5}$, $\frac{7}{9}$, $\frac{5}{6}$, and 1.115 inches long.

68. Construct a square equal to $\frac{2}{3}$, the area of the above triangle.

69. Find by construction a fourth proportional to three lines respectively $2\frac{1}{7}$, $3\frac{25}{30}$, and $1\frac{5}{7}$ inches long.

70. Construct a square 1.3 inches high, and inscribe in it an equilateral triangle containing a circle tangent to its sides; describe about the square a triangle having two of its angles 57° and 46°, and describe a circle passing through its angles.

71. In an equilateral triangle 2.7 inches high inscribe a square, and also describe a square around it.

72. Divide a straight line 5.67 inches long into 12 equal parts, and through the points of division draw parallel lines .25 inch apart, inking them alternately dotted and continuous.

73. Draw a straight line AB 3.1 inches long, assume a point C .87 inches above B in such a position that a line joining BC would be perpendicular to AB. Describe a circle passing through C, and touching B at a point .78 of an inch from A.

74. Describe a circle of 2.47 inches diameter, and in it inscribe a triangle having angles of 49, 78 and 53°. In the triangle inscribe a square.

75. An isosceles triangle having a vertical angle of 32° stands on a base of 2.55 inches: transform its shape by lowering its vertex to within one inch of its base.

76. Find two points, B and C, that will be at right

angles to two other points, A and D, 2.3 inches distant from each other.

77. The angle C at the centre of a polygon being given, (say 72°), construct on a base AB, 1.5 inches long, the polygon to which it belongs.

78. Divide a line 6.7 inches long, according to the ratios of another line divided severally into $\frac{2}{3}$ of .76 of $\frac{5}{9}$ of $\frac{7}{8}$ of $\frac{180}{495}$ of .85 of a line 5.2 inches long. (Use sector.)

79. Construct a triangle having its sides AB and BC = to 3.2 and 3.7 inches long and the angle ACD = 47° 15′. Ascertain and figure the length of the third side and the values of the remaining angles.

80. Draw two lines AB and AC forming at A an angle of 39°, and describe a circle of .86 inches radius, tangent to both lines.

81. Draw two straight lines 2.7 inches apart and unite them by two curves of contrary flexures, having radii of .82 and 1.37 inches respectively, in such a manner that each of them shall touch the other, and both be tangent to the two straight lines.

82. Draw a line 1.8 inches long, and on it describe a segment containing an angle of 57°.

83. The base AB of a triangle ABC = .785 of a line 2.865 inches long. The angle ABC = 47° 45′ and the angle BAC = 71° 15′. Find very accurately the length of the two other sides. On AC as a base, construct a regular pentagon, and on BC construct a regular nonagon: reduce the whole figure to a square of equal area.

N.B.—In solving this problem (which is intended as a recapitulation of the use of instruments) find the length of the line 2.865 by the diagonal scale on the protractor and its fraction .785 by the line of lines on the sector. The

angles are to be obtained by the lines of chords, and the polygons are likewise to be constructed by the lines of polygons on the same instrument.

PRINCIPAL UNITS OF LINEAL MEASURES REDUCED TO ENGLISH INCHES, FEET, YARDS, MILES, ETC.

	UNIT.	INCHES.	FEET.	YARDS.	MILES.	..
AUSTRIA .	Zoll (12 linien) .	-	.08640	.02880		
"	Fuss or Schuh .	-	1.03704	.34568		
"	Elle . . .	-	-	.85289		
"	Klafter (6 fuss) .	-	-	2.0741		
"	Ruthe (10 fuss) .	-	-	3.4568		
"	Meile . . .	-	-	5863.3	3.3312	
"	Meile Geographische	-	-	8100.8	4.6026	
BADEN .	Fuss (foot) . .	-	.9842	.32806		
BAVARIA .	Fuss . . .	-	.95751	.31917		
"	Ruthe (10 fuss) .	-	-	3.1917		
BERNE .	Pied (12 pouces)	-	.96216	.32072		
"	Anne . . .	-	-	.59557		
"	Perche . . .	-	-	3.2064		
BELGIUM .	Fuss (11 zolle) .	-	.90466	.30155		
"	Antwerp foot .	11 240	.9367			
"	Brussels foot .	11.450	.9542			
"	Elle . . .	-	-	.74845		
"	Verge . . .	-	-	4.9255		
"	Meile . . .	-	-	4860.833	2.7641	
CHINA .	Tché (foot) . .	-	1.05	.3500		
"	mile . .	-	-	609.	.3456	
DENMARK .	Foot . . .	12.357	1.02975	.34325		
"	Ell . . .	-	2.05950	.68650		
ENGLAND .	Inch . . .	-	.08650	.02777		
"	Foot . . .	-	1.0000	.33333		
"	Yard . . .	-	3.000	1.000		
"	Ell . . .		-	1.2500		
"	Fathom . .	-	-	2.000		
"	Pole *or* Perch .	-	-	5.5000		
"	Chain (100 links) .	-	-	22.000		
"	Furlong . .	-	-	220.000		
"	Mile (statute) .	-	-	1760.000	1.000	
"	Mile, Geographical and Nautical	-	-	2025.200		
		-	-			
"	League . .	-	-	5280.000	3.000	
FRANCE .	Millimetre . .	-	.0033	.0011	-	
"	Centimètre . .	-	.0327	.0109	-	
"	Decimètre . .	-	.3279	.1093	-	New
"	Mètre . . .	-	-	1.0936	-	measure.
"	Kilomètre . .	-	-	1093.63	.62138	
"	Myriamètre . .	-	-	10936.33	6.2138	
"	Pouce (12 lignes) .	-	.08864	.02955	-	
"	Pied . . .	-	1.06571	.35523	-	
"	Toise (6 pieds) .	-	-	2.13142	-	Old
"	Brasse marine (5 pieds)	-	-	1.77618	-	measure.

	Unit.	Inches.	Feet.	Yards.	Miles.	..
France .	Lieue deposte (2000 toises	-	-	4262.84		
"	Lieue Marine (.05 degree)	-	-	6275.6	3.4519	
"	Mille Marin (.333 lieue) =1 minute	-	-	2025.2	1.1506	
Cologne .	Fuss . . .	10.830	.9025	-		
Heidelburgh	Fuss . . .	10.96	.9133	-		
Frankfort-on Maine	Fuss . . .	-	.9399	.3133		
"	Elle . . .	-	1.7703	.5901		
Hamburg .	Fuss . . .	-	.9399	.3133		
"	Elle . . .	-	1.8798	.6266		
Hanover .	Fuss . . .	-	.9579	.3193		
"	Meile . . .	-	-	7442.	4.2287	
Holland .	Fuss . . .	-	.9288	.3096		
"	Meile . . .	-	-	6404.12	3.6387	
India .	Covido, Hath or Hasta	-	-	.75		
Bengal .	" mile . .	-	-	2000.	1.1364	
Mecca .	Covid . . .	-	-	.5		
Arabia .	" mile . .	-	-	2146.	1.2193	
Milan .	Miglio . . .	-	-	1808.81	1.0277	
Bologna .	" (foot) .	14.928	1.244	-		
Florence .	" (foot) .	12.79	1.0658	-		
Italy .	Miglio . . .	-	-	2025.	1.1508	
Norway .	Mile . . .	-	-	12182.	6.9216	
Naples .	Palma . . .	-	.8628	.2876		
"	Canna . . .	-	-	2.3008		
"	Miglio . . .	-	-	2018.	1.1468	
Persia .	Parasang . .	-	-	6299.04	3.579	
Piedmont .	Miglio . . .	-	-	8527.	4.8445	
Portugal .	Pie . . .	12.96	1.08266	.3608		
"	Palmo . . .	-	.7171	.2390		
"	" mile . .	-	-	2250.	1.2787	
Prussia .	Fuss (Rhenish foot)	-	1.0297	.3432		
"	Elle . . .	-	-	.7293		
"	Schritt . .	-	-	.8234		
"	Klafter *or* Faden (6 feet)	-	-	2.0594		
"	Ruthe (12 feet) .	-	-	4.1188		Decimally divided.
"	Meile . . .	-	-	8272.	4.7	
Malta .	Piede (foot) .	11.17	.9308	-		
Rome .	Piede . .	-	.9665	.3222		
Russia .	Archine . .	-	-	.7777		
"	Sachine . .	-	-	2.3332		
"	Verst . . .	-	-	1166.6	.6628	
"	Foot . . .	13.75	1.1458	-		
Saxony .	Fuss . . .	-	.9294	.3098		
"	Meile . . .	-	-	9923.326	5.6382	
Spain .	Pulgado . .	-	.0761	.0254		
"	Palmo . . .	-	.6849	.2283		
"	Pie (.9275) (Castillian) (12 Pulgados)	11.13	.9132	.3044		

	Unit.	Inches.	Feet.	Yards.	Miles.
Spain	Vara (3 pies)	-	-	.9132	
"	Misle	-	-	1522.	.8648
Sweden	Foot (10 inches)	11.69	.9742	.3246	
"	Alner	-	1.948	.6493	
"	Mile	-	-	11688.	6.6412
Turin	Piede	-	1.1237	.37458	
"	Auna	-	1.9714	.65714	
"	Pertica	-	-	6.742	
"	Genoa Palm	9.808	.8173	-	
Turkey	Berri	-	-	1823.08	1.0358
Tuscany	Piede (Geographical)	-	1.9094	.6364	
"	Piede (common)	-	1.7985	.5995	
Venice	Piede	-	1.1410	.3803	
"	Auna	-	2.0892	.6964	
Westphalia	Stunde (.5 mile)	-	-	6075.52	3.4562
Wurtemburg	Fuss	-	.9399	.3133	
Zurich	Pied	-	.9888	.3296	
Ancient Egypt	Cubit	-	1.4764	.4921	
" Greece	Cubit	-	1.4764	.4921	
" "	Stadium	-	-	196.85	
Rome	Stadium	-	-	201.29	.1143

Scales.

Although I have already explained the general method of constructing plain and diagonal scales, page 6, I will not apologise for referring again to that subject, practice having convinced me that it very often may advantageously bear repetition.

A scale is a conventional object intended to represent artificially certain dimensions bearing some stated proportions to some given real magnitudes.

In Geometrical drawing the scales most generally used are the plain and the diagonal scales.

When a given space on a map or plan represents some known distance which has to be represented by a scale not requiring very minute subdivisions, a plain scale will suffice.

Let us suppose, for instance, that the space of one inch upon a map should represent a length of 10, 15, or 20 feet, yards or miles, &c. and not require any more minute

subdivision, we should construct a plain scale, because it is easy to subdivide the space of one inch into, or in the ratio of 10, 15, or 20 equal parts.

Should we, however, require to subdivide the feet into inches, the yards into feet, or the miles into furlongs, we should be compelled to construct a diagonal scale, because we would find it impossible, or very difficult, to obtain the scale by any other means.

Practice.

Construct a scale in the ratio of 12 feet to 1 inch. Let 70 feet be required.

Then, 12 feet is to 1 inch as 70 feet is to 5.833 inches.

Draw a line 5.83 inches long, divide it into seven equal parts, or primary divisions, each of which will represent 10 feet. Subdivide the left hand primary division into 10 equal parts (or single feet.) (*Euc.* 2. VI.)

Draw a thick second line at $\frac{4}{80}$ of an inch below, and parallel to the first, and number, as an example, Fig. 1, Pl. XIV. Print before the scale, "scale of," and at the further extremity, "feet," or whatever may be the denomination of the units intended to be represented by the scale.

As in this case the space of one inch represents 12 feet of real dimensions, the proportion between the scale and the original distance will evidently be the same as between one inch and as many inches as are contained in 12 feet, therefore 12 feet reduced to inches $= 12 \times 12 = 144$ inches, which are represented by one inch, and consequently give the representative fraction of the scale $\frac{1}{144}$.

Representative Fraction.

A representative fraction, therefore, determines the proportion that exists between the length of the scale and the original dimension which it represents. In other words, it

shows the proportion between an inch, a foot, a yard, a mile, or any other denomination as represented on paper, with a real inch, foot, yard, or mile, &c. in real length on the ground. The representative fraction is usually placed over the scale. The scale is now completed, and ***all plain scales*** are constructed after this method. The secondary subdivisions varying only with the number of units represented by one primary division. Thus, if one of the primary divisions represents one foot to show inches, the secondary subdivision will be subdivided into 12 equal parts. If it represent fathoms, to show feet it will be subdivided into 6 parts. Furlongs to show chains, into 10 parts; miles into furlongs, into 8 parts; yards into feet, into 3 parts, &c.

Should we require a scale of 120 feet to 1 inch. It is evident that we could not divide one inch accurately into 120 parts by the method just explained. We must, therefore, construct a ***diagonal scale.*** Let us assume 70 feet then as in the former scale $120 : 1 :: 700 : X = 5.833$.

Divide a line 5.83* into 7 equal parts, each representing 100 feet, and subdivide the first primary division into 10 equal parts, or secondary divisions, each representing 10 feet. Draw below this scale line .10 other lines parallel and equidistant from each other (any space will do, but one-tenth of an inch apart is usually found a convenient distance, the size of the scale, however, being the best guide to follow.) From the first secondary subdivision to the left of the lowest line draw a diagonal line to the upper intersection of the highest horizontal line with the left hand extremity of the scale, and from each of the other subdivisions of the base draw lines parallel to this, Fig. 2, Pl. XIV., and number the scale as in example.

* In practice it is seldom necessary to go beyond two or three places of decimals. Two places when the third decimal is less than 5; three places when it is more.

The primary divisions, as we have said, will represent hundreds of feet. Each secondary division will represent spaces of 10 feet, each of which will again be subdivided into single feet by the diagonals determining the tertiary subdivisions, which are marked accordingly.

Draw a base line rather wider apart than the rest, and likewise thicker. Print "scale of —— feet," and mark the representative fraction as before.

All diagonal scales are constructed after this method.

Like the plain scales, they only vary in their subdivisions and dimensions. Scales should always be divided decimally, and consequently, except when very large, completed to some power or multiple of 10. They vary in length according to the size of the drawing, map, or plan they are to accompany, therefore no definite rule can be given on that score. The length of the scale must depend on the nature of the distances that are likely to be required. Usually, however, we find for common purposes a length varying from 5 to 10 inches most convenient.

In questions involving tho construction of scales, two things are usually given. First, a certain given real distance; and secondly, a certain space on the plan or map assumed to represent that distance artificially, as,

A real distance of 25 miles is represented on a map by a space of 1.5 inches. It is easy (that space being so small) to assume some distance divisible by 10 that will complete the scale to some convenient length between 5 and 10 inches. Here we have the simple proportion.

25 miles : 1.5 inches : : 100 miles : X = 6 inches, or
25 „ : 100 „ : : 1.5 : X =

hence the rule. Place in the

1st TERM.—The real distance given, reduced to the denomination of the scale required.

2nd Term.—The space on paper representing the distance given in the 1st term.

3rd Term.—The whole distance, forming the length of the scale required, or assumed, always some power of 10.

4th Term.—Will give results on paper that will represent the whole length of the scale.

Examples.

1. The distance between London and Egham is 18 miles, and measures on a map 3.7 inches. Complete the scale—to 30 miles. Then $18 : 3.7 :: 30 : X = 6.166$.

A length of 6.17 nearly, divided into 3 equal primary divisions, will each show a space of 10 miles, and the first left hand division subdivided into 10 secondary equal parts will show single miles. The representative fraction of the scale will be $\frac{1}{18 \times 1760 \times 36 \div 3.7}$. (Fig. 3, Pl. XIV.)

2. The distance between London and Chatham is 30 miles, and is represented on a map by a space of 18.6 inches, show 10 miles and fnrlongs (1 mile = 8 furlongs.)

$30 : 18.6 :: 10 : X = 6.2$. A line 6.2 inches long, divided into 10 parts, will show miles, and the first primary division, subdivided into 8 parts will each show $\frac{1}{8}$ of a mile, or a furlong. The representative fraction will be $\frac{1}{10 \times 1760 \times 36 \div 6.2}$. (Fig. 4.)

3. On a plan, 4 feet 6 inches are represented by a space of 2.5 inches, complete the scale to 10 feet and show inches (1 foot = 12 inches.)

4.5 feet : 2.5 inches : $10 : X = 5.55$. (Fig. 5.)

The first primary division, representing one foot, is subdivided into 12 parts, each representing one inch. The representative fraction $= \frac{1}{10 \times 12 \div 5.55}$.

4. On a plan, a space of 3 inches represents 9 fathoms. Complete the scale to 20 fathoms, and show feet, (1 fathom = 6 feet.) (Fig. 6.) 9 : 3 : : 20 : X = 6.66. Representative fraction $\frac{1}{20 \times 6 \times 12 \div 6.66}$.

Should we be undecided as to what will be the whole length of the scale when completed, we might also construct it thus :—

Let a space of 2.5 inches represent, say 17 feet.

Let us obtain first the value of one foot, and then repeat it on the scale line as many times as desirable. Then 17 feet : 2.5 : : 1 : X = .14705, or 1.4705 for 10 feet. As on account of the long decimals frequently obtained by this method the scale cannot be constructed to so great a degree of accuracy, the method first explained is preferable.

I will refer hereafter to another method of constructing scales Geometrically, but trust that the preceding examples will suffice to guide in the construction of plain scales, the learner bearing in mind that the first left hand primary division is always subdivided into as many secondary units as are equivalent to the value of one of the primary divisions.

Exercises on Representative Fractions.

Give the representative fraction of a scale on which 1 mile is represented by 1 inch, 1 mile = 1760 yards.

Of a river 1475 yards broad, represented on a map by a space of 4 inches, 1 yard = 36 inches.

Also when 147 feet are represented by 3.5 inches, 1 foot = 12 inches.

When 26 miles are represented by 3.4 inches, 1 mile = 63360 inches.

When 9 chains are represented by 1.3 inches, 1 chain = 22 yards.

When 4 fathoms are represented by 5.9 inches, 1 fathom = 6 feet.

When 3 Russian versts are represented by 2.72 inches, 1 verst = 1167 yards.

When 2 Austrian miles are represented by 4.76 inches, 1 Austrian mile = 3.3312 English miles.

Scales are often given by their representative fraction, as,—

1. Required a scale of yards $\frac{1}{187}$, that is to say, one yard equals 187 yards. If we reduced both terms of this fraction to inches, we would have $\frac{36}{6732}$ inches, or 36 inches = 187 yards; therefore 36 inches : 187 yards :: 1 : X = 5.2 yards nearly. A space of one inch would then represent 5.2 yards. As both terms of the fraction are of the same denomination, it would precisely give the same result to consider one inch on the paper equal to 187 inches, or 5 yards 7 inches.

Therefore the scale might be completed thus, say to 30 yards:—

$$5.2 \text{ yds.} : 1 :: 30 : X = 5.77 \text{ nearly.}$$

2. Required a scale of miles whose representative fraction is $\frac{1}{614373}$, then 1 inch equals 614373 inches, and as one mile contains 63360 inches, we find that $\frac{614373}{63360} =$ 9.696543 miles represented by one inch; then,

$$\begin{array}{cccc} \text{Miles.} & \text{Inch.} & \text{Miles.} & \\ 9.6965 \;. & 1 \;:: & 60 \;: & X = \end{array}$$

3. Required a scale of feet whose representative fraction is $\frac{3}{500}$, then 3 feet or 36 inches = 500 feet ∴ 500 : 36 :: 100 : X = 7.2.

4. Required a scale of yards whose representative fraction = $\frac{1}{144}$; then, $\frac{144}{36}$ = 4 yards to 1 mile.

5. Construct a scale of feet whose representative fraction is $\frac{1}{48}$.

6. Construct a scale of yards whose representative fraction is $\frac{1}{72}$.

7. Construct a scale of miles whose representative fraction is $\frac{1}{63360}$.

8. Construct a scale of fathoms whose representative fraction is $\frac{1}{216}$.

9. Construct a scale of yards whose representative fraction is $\frac{1}{10000}$.

10. Construct a scale of chains whose representative fraction is $\frac{1}{800}$.

11. Construct a scale of fathoms whose representative fraction is $\frac{3}{700}$.

12 Construct a scale of Russian versts whose representative fraction is $\frac{3}{978000}$. 1 verst = 1166.68 English miles.

Diagonal Scales.

Construct a scale of 9 miles to 1.3 inches, showing furlongs diagonally—

$$9 : 1.3 :: 4 : 5.67$$

5.67 inches divided into 4 primary divisions, will show spaces of 10 miles, the first primary division into 10 parts will show single miles; 8 lines drawn parallel to the scale line (as many as there are tertiary units), and divided by the diagonal method, will show furlongs. (Fig. 7.)

2. Construct a scale of 10 feet to 1.5 inches, showing inches. $10 : 1.5 :: 40 : X = 6$ inches.

As the tertiary divisions will be inches, and as a foot contains 12 inches, twelve lines drawn parallel to the first will be required. (Fig. 8.)

3. A scale of 18 fathoms to 1 inch to show feet—

$$18 : 1 :: 100 : X$$

As one fathom equals 6 feet, 6 lines parallel to the first will be required.

4. A scale of miles 76 to 1.3 inches. 76 : 1.3 :: 500 : X = 8.29 nearly. Five primary divisions will show hundreds of miles, 10 secondary divisions will show spaces of 10 miles, and 10 lines parallel will show single miles.

Comparative Scales

Are scales representing precisely the same distance and consequently having the same representative fraction as other scales, but under different denominations; thus, whether we require scales of 2 fathoms to 1 inch, or of 12 feet to 1 inch, or of 4 yards to 1 inch, or of 144 inches to 1 inch, those scales will be comparative because their representative fractions, $\frac{1}{144}$, are the same: 2 fathoms being equal to 144 inches, 12 feet = 144 inches, and 4 yards = 144 inches; the space of 1 inch on the scale representing in every case 144 real inches.

Should we, again, require—

a scale of yards, 1760 yards to 1 inch;
a scale of miles, 1 mile to 1 inch;
a scale of furlongs, 8 furlongs to 1 inch;
a scale of chains, 80 chains to 1 inch;

the representative fraction $\frac{1}{63360}$ would be the same because all those different denominations or scales reduced to inches would give that number of inches represented by one inch, therefore all those scales would be comparative to each other.

Having a scale on which a space of 2.7 inches represent 1 mile, and whose representative fraction will naturally be $\frac{1}{1760 \times 36 \div 2.7}$, I require a scale of yards comparative to it; it is evident that the scale will only change its denomination, because 2.7 will represent 1760 yards instead of being called 1 mile.

That scale, therefore, will be 1760 : 2.7 :: 4000 : X = 6.18. It follows that in order to obtain a comparative scale we have only to change the denomination of the given scale into that of the scale required, which is performed by simple multiplication or division, as the case may be; thus, suppose that on a map a space of one inch represents 5 yards, show a comparative scale of feet. Here reduce the yards into feet, $5 \times 3 = 15$ feet, that scale will naturally become a scale of 15 feet to an inch, with representative fraction $\frac{1}{5 \times 3 \times 12}$, and conversely, should it be required to convert a scale of 15 feet to an inch into yards, $\frac{15}{3}$ would give 5 yards, and the representative fraction $\frac{1}{15 \times 12}$ would be the same as above.

Examples.

1. Reduce 40 yards to Russian archines. 1 archine = .7777 of a yard.

$$\frac{40}{.7777} = 51.433 \text{ archines.}$$

2. Reduce 33 metres to Greek cubits. 1 cubit = .45 of a metre.

$$\frac{33}{.45} = 73.33 \text{ cubits.}$$

3. Reduce 72 feet to French metres. 1 metre = 3.27 feet.

$$\frac{72}{3.27} = 22.018 \text{ metres.}$$

4. Reduce 73 Spanish palms to English feet. 1 palm = .684 English foot.

$$.684 \times 73 = 50.132 \text{ feet.}$$

5. Reduce 57.61 metres into English yards. 1 yard = 1.09363 metres.

$$\frac{57.61}{1.09363} = 52.679 \text{ yards.}$$

6. Reduce 12 feet to Roman palms. 1 palm = .735 of 1 foot.

$$\frac{12}{.735} = 16.32 \text{ palms.}$$

7. Reduce 27 Roman palms to English feet.

$$27 \times .735 = 19.845 \text{ feet.}$$

8. Reduce 168 yards to chains. 1 chain = 22 yards.

$$\frac{168}{22} = 7.636 \text{ chains.}$$

PRACTICE.

Reduce 64 Spanish palms to English feet. 1 palm = .684 of English foot.

Reduce $2\frac{1}{8}$ miles to paces. A mile assumed to contain 2250 paces.

Reduce 686 Neapolitan cannae to English yards. 1 canna = 2.3008 English yards.

Reduce $17\frac{1}{2}$ Austrian miles to English miles. 1 Austrian mile = 3.3312 English miles.

Reduce 300 Russian sachines to English yards. 1 sachine = 2.332 yards.

Reduce 8000 alners to English feet. 1 alner = .6493 of an English yard.

EXAMPLES OF COMPARATIVE SCALES.

1. Construct a scale of 5 chains to 1.5 inches, and a comparative scale of yards to it. 1 chain = 66 feet, or 22 yards.

For the first scale, 5 chains : 1.5 :: 20 : X = 6. 6 inches divided into two parts, and the first division subdivided into 10 parts, will give the scale of chains.

For the second scale, reduce the chains to yards,
$5 \times 22 = 110$ yards. $\therefore$ 110 : 1.5 :: 500 : X = 6.81.

2. On a Spanish plan, 47 palms are represented by a

distance of 2.3 inches; construct a scale of English feet comparative to it. 1 palm = .684 English feet.

(.684 × 47 = 32.148 English feet) ∴ 32.148 : 2.3 :: 100 : X = 7.185.

3. Construct a scale of 2 inches to 1.5 mile, upon which hundreds of paces can be measured, assuming a mile to contain 2260 paces.

2260 × 1.5 : 2 :: 10000 : X = 5.9.

5.9 inches divided into 10 parts will show thousands of paces, the 10 secondary divisions will show hundreds, and the diagonal tertiary divisions will determine spaces of 10 paces.

4. On an Italian plan, 4 inches represent 325 cannae; show a comparative scale of English yards. 1 canna = 2.3008 English yards.

(2.3008 × 325) : 4 :: 1000 : X =

5. The distance between two points three Austrian miles apart is represented on a map by a space of 6.67 inches. Construct a scale of English miles and chains. 1 Austrian mile = 3 3312 English miles. 1 English mile = 80 chains.

3.3312 × 3 = 9.9936 English miles ∴ 9.9936 : 6.67 :: 10 : X =

6. Construct a scale of 1.45 inches to a mile, and show furlongs; give a comparative scale of chains.

First scale—1 mile : 1.45 :: 5 : X = 7.25. (Plain scale.)

Second scale—$\frac{1760}{22}$ = 80 chains ∴ 80 : 1.45 :: 400 : X = 7.25. (Diagonal.)

7. Construct a scale of $\frac{1}{190080}$ to represent English miles and show furlongs. $\frac{190080}{36}$ inches = $\frac{5280}{1760}$ yards = 3 miles.

∴ 3 : 1 :: 20 : X =

8. To the latter distance, construct a comparative scale of Turkish berri. 1 berri = 1828.08 yards.

Then $\frac{20 \times 1760}{1828.08}$ = the number of berri in 20 miles.

$\therefore$ 19.239 berri : 6.66 :: 20 : X

9. On a Spanish plan, 46 palms are represented by a space of 2.3 inches; give a comparative scale of English yards. 1 palm = .684 of an English foot.

$$\left(\frac{46 \times .684}{3}\right) : 2.3 :: 30 \text{ yds.} : X =$$

10. On a Swedish map, 6000 alners are represented by a space of 5 inches; give a comparative scale of English feet. 1 alner = .6493 of an English yard.

$(6000 \times .6493 \times 3) = 11687.4 : 5 :: 15000 : X$

11. Required, a scale of 8 feet to 1 inch, and a comparative scale of French decimetres. 1 metre = 1.0936 yards.

First scale, 8 ft. : 1 :: 50 : X = 6.25.

Second scale, $\frac{1.0936 \times 8}{3} = 2.9162\dot{6}$ metres : 1 :: 20 : X = 6.85.

12. Construct a plain scale of yards, 290 yards to 1.9 inches, and also a corresponding scale of Russian sachines. 1 sachine = 2.332 yards.

First scale, 290 : 1.9 :: 1000 : X

Second scale, $\frac{290}{2.332}$: 1.9 :: 500 : X.

13. Construct a scale of $\frac{1}{300}$, 50 yards long, and a comparative scale of Russian archines. 1 archine = .7777 of a yard.

1 yard = 300 yards $\therefore$ 36 inches = 300 yards.

then 300 : 36 :: 50 : X = 6.

Second scale, $\frac{300}{.7777} = 387.1672$ archines : 36 :: 60 : X

14. Draw scales of $\frac{4}{700}$ to represent English feet, French metres, and Greek cubits.

$\left.\begin{array}{l}\text{1 French metre} = 3.27 \text{ feet} \\ \text{1 Greek cubit} = .45 \text{ of a metre}\end{array}\right\}$ show 100 feet, 30 metres, and 50 cubits.

First scale, $\frac{4}{700}$ of 100 feet $= \frac{4}{7}$ of 1 foot $= 6.85713$, or 6.86 inches nearly, then 6.86 will equal the length of the first scale.

Second scale, 30 metres $\times$ 3.27 feet $= 98.1$ feet $\therefore$ $100 : 6.85 :: 98.1 : X = 6.72$ inches.

We said that 1 Greek cubit $= .45$ of 1 metre $\therefore$ 1 cubit : .45 metre :: 60 : X $=$ 27 metres.

Third scale, $\therefore$ 30 m. : 27 :: 6.72 : X $=$ 6.081 length of 60 cubits.

15. Two plans are drawn to different scales, the first on a scale of 1.75 inches to 1 mile, and the second on a scale of 1.75 miles to 1 inch; show the length of the line which for each plan would represent a distance of 3.5 miles, and give both representative fractions.

First scale, 1 mile : 1.75 inches :: 3.5 : X $= 6.125$, rep. fraction $\frac{1}{1760 \times 36 \div 1.75}$.

Second scale, 1.75 miles : 1 inch :: 3.5 : X $= 2$, rep. fraction $\frac{1}{1.75 \times 1760 \times 36}$.

Although the arithmetical mode of determining the dimensions of scales is the most correct, there are other methods of constructing them which may sometimes prove convenient. For instance, should we require a scale of 40 yards in length, in the ratio of 7 yards to 1 inch, the result would be 7 : 1 :: 40 : X $=$ 5.714285 inches, a very long decimal, which it would be impossible to obtain by any scale: in such a case the Geometrical construction would be preferable: to obtain it,

Draw an indefinite scale line about 5 or 6 inches long, and upon it from its left extremity determine one inch.

Divide that inch geometrically into 7 parts, and add 3 more subdivisions to complete to 10; this will give the first primary division subdivided into 10 yards—take that length in your compass and step it 3 times more on the scale line, which is to be completed as already explained.

Of course practice must decide which method is the most appropriate in any given case. Comparative scales may likewise be often conveniently divided in the same manner.

Suppose a space of 3.7 inches on a map represents a distance of 22 miles, complete the scale to 30 miles (Fig. 9, Pl. XV.) and construct to it a comparative scale of Russian versts. 1 mile = 1760 yards. 1 verst = 1167 yards.

On the scale line AB make AD = to 3.7 inches, and from A draw any diagonal line of construction AX.

From any scale of equal parts, say 20 to 1 inch, take 22 units in your compass and transfer them on AC: from the same scale take 30 equal parts and from A lay them on the prolongation AC in AX, join C 22 and D 3.7, and draw X 30—B parallel to it.

The length AB will be that of the scale representing 30 yards, which, divided into 3 parts, and the first division into 10 parts, will give the scale, to be completed as usual.

For the comparative scale of Russian versts (Fig. 10, Pl. XV.) make AB equal to 10 units of the subdivisions of the first scale, and draw the diagonal AC.

From any scale of equal parts take 1760 and mark it on A—C A—1760, from the same scale take 1167, and mark on the same line A—1167. Join 1760—B, and draw 1167—D parallel to it.

As 1167 yards is to 1760 yards, so is one verst to one yard; or, so are 10 versts to 10 yards. Complete the scale to the required length and finish it as usual.

Most other scales can be obtained in the same manner, but they are never so minutely accurate as when they are constructed arithmetically.

Verniers

Are small moveable scales attached to the graduated limbs of astronomical and surveying instruments, beam compasses, barometers, &c., and so constructed as to enable us to read with the greatest degree of accuracy the measured distances, angles or degrees obtained by the instrument to which they belong; although they may in some cases be used instead of diagonal scales, they are seldom used in drawing. A knowledge of them and of their construction is nevertheless important.

General rule for the construction of Verniers.

Take in the compass a space equal to one unit less than that of the whole length of one primary division of the scale, subdivide this length into as many units as are contained in the whole primary division of the scale required: therefore, to construct a vernier to read inches, tenths and hundredths of inches, divide a line into any number of inches, and subdivide each inch into 10 equal parts; take in the compass .9 of an inch, and divide that space into ten equal parts, each of these parts will naturally be $\frac{1}{10}$ smaller than each tenth of an inch, and that vernier being shifted along the scale will point out by the coincidence of two of their separate subdivisions, 10ths and 100ths of an inch. (Fig. 11, Pl. XVI.)

Similarly for a scale of degrees into minutes—1° equals 60′.

The degrees divided into 6 parts will each equal 10 minutes, nine of these parts subdivided into 10 will show $\frac{1}{60}$ of a degree or 1 minute. (Fig. 12.)

A scale of fathoms into feet—1 fathom equals 6 feet.

A length of 5 fathoms divided into 6 parts will show single feet. (Fig. 13.)

A scale of miles and furlongs—1 mile equals 8 furlongs.

A length of 7 miles divided into 8 parts will show single furlongs. (Fig. 14.)

A scale of feet and inches.

Eleven feet divided into 12 parts will show $\frac{1}{12}$ of one foot, or inches. (Fig. 15.)

Circular instruments are generally subdivided into 360 equal parts or degrees, each degree being equal to 60 minutes.

Construct a portion of a circular limb to read degrees and minutes.

With any convenient radius, draw four concentric arcs .2 of an inch apart, so as to form a sector of 40 degrees—or of any other required length,—divide that space of 40 degrees into 4 equal parts, reaching from the outer to the inner circle; divide each of these four parts of 10 degrees into 10 equal parts or single degrees, reaching from the middle to the outer circle; subdivide each degree on the outer circle into 3 equal parts, each showing 20 minutes. The length of 19 of those parts subdivided in 20 equal parts will show spaces of a single minute. (Fig. 16.)

Similarly to construct a vernier to read to 5 minutes.

Eleven divisions of the limb equal 12 on the vernier.

Vernier to read to $\frac{1}{2}$ minutes: 29 divisions on the limb equal 30 on the vernier.

EXAMPLES FOR PRACTICE.

1. Draw a scale whose representative fraction is $\frac{1}{12}$ to show feet and inches.

2. Draw a scale whose representative fraction is $\frac{1}{36}$ to show yards and feet.

3. Draw a scale whose representative fraction is $\frac{1}{63360}$ to show miles and furlongs.

4. A scale of 160 fathoms to 9 inches.

5. A scale of 63 yards to 1.57 inches.

6. A scale of 5.3 miles to 1.31 inches.

7. A scale of 18 feet to 1.42 inches.

8. A scale of 4 feet 5 inches to 1.89 inches.

9. A scale of 8.9 miles to 4.785 inches.

10. A scale of $\frac{1}{16}$ to show feet and inches.

11. A scale of $\frac{1}{28}$ to show feet and inches.

12. A scale of $\frac{1}{187}$ to show fathoms and feet.

13. A scale of $\frac{1}{155376}$ to show miles and furlongs.

14. A scale of $\frac{1}{5}$ of an inch to a mile to show miles.

15. A scale of 18 miles to 2.15 inches.

16. A scale of 2.77 yards to 1.895 inches.

17. A scale of 485 miles to 4.687 inches.

18. A scale of 87 fathoms to 5.1 inches.

19. A scale of 47 yards to 3.2 inches showing feet.

20. A scale of 38 miles to 4.3 inches showing furlongs.

21. A scale of 5 feet to 1.7 inches showing inches.

22. A scale of $\frac{4}{5}$ of an inch to 1 foot 6 inches.

23. A diagonal scale of 1 foot to .87 inch showing $\frac{1}{2}$ inches.

24. A diagonal scale of 4 miles to 1.2 inches showing furlongs.

25. A diagonal scale of 9 fathoms to .55 of an inch showing feet.

26. Of 19 yards to .56 inch showing feet.

27. Of $\frac{1}{386}$ to show yards.

28. Of 9 miles to 1.3 inches to show furlongs and chains.

29. Of one foot to show 1000th of a foot.

30. Of 4 miles 588 yards to 3.1 inches.

31. On a plan the distance between two points 4.27 inches apart represent 4587 yards: construct a scale of miles, furlongs and chains.

32. Draw a scale to measure feet on a plan on which the space between the two points 1.5 inches apart is $\frac{1}{287}$ of the real distance.

33. Construct a scale of metres comparative to one of 17 English yards to 1.2 inches 1 metre = 39.34 inches.

34. The distance between London and Woolwich is 9 miles: construct the scale of a plan on which that distance is represented by .65 inches, and show furlongs and chains.

35. Construct a scale of $\frac{1}{5000}$ for a regular enclosure, 1680 feet long and 1350 feet broad.

36. Construct a scale of yards 5 inches to 6 furlongs.

37. A regiment drawn up in order of battle occupies a front of 800 feet, and measures on the plan 29 inches: construct the scale of the plan to show poles and yards. N.B. 5.5 yards = 1 pole.

38. Construct a scale of fathoms $\frac{1}{20,000}$.

39. On a plan $\frac{1}{5}$ of a mile occupies a space of 2.7 inches: construct a scale of yards to it.

40. On a map 183 miles occupy a space of 14.3 inches: construct a scale to show furlongs.

41. Construct a scale of paces for a plan, draw to a scale of $\frac{1}{1567}$, assume the pace to equal 2.67 feet.

42. Construct a scale of $\frac{1}{9050}$ to represent yards.

43. Draw a scale of paces 11 inches to a mile: supposing 25 paces to equal 1 chain: 1 chain = 66 feet.

44. Construct a scale of Milan miles: 5 to 1 inch. 1 Milan mile = 1.0277 English miles.

45. Draw a scale of 4 inches to a mile, to show furlongs and spaces of 50 feet: give its representative fraction.

46. Draw a scale of $\frac{3}{978900}$ to show English miles and Russian versts. 1 verst = 1166.68 English yards.

47. Construct a scale of paces for a plan drawn to a scale of $\frac{1}{3298}$: the pace being assumed to equal 2.5 feet.

48. On a plan 18000 paces measure 22.3 inches: show spaces of 10 paces diagonally.

49. Draw a scale of 14 yards to 1.1 inch, and a comparative scale of metres—1 metre = 39.34 inches.

50. On a map $\frac{4}{5}$ of an inch represent 13.68 yards: show 100 yards and the representative fraction.

51. A table 14 feet 6 inches long is represented by a space of 5.2 inches: make a diagonal scale showing $\frac{1}{4}$ inches—to the drawing.

52. Construct a scale of $\frac{1}{200}$ to represent Belgian and Chinese feet.

1 Belgian foot = .90466 of an English foot.
1 Chinese foot = .8616 of an English foot.

53. Construct a scale of $\frac{2}{300}$ to show English yards and Russian archines. 1 archine = .7777 of an English yard.

54. Construct a scale of chains 1 inch to 1 mile.

55. On a plan, 4.65 inches, represent 13.5 miles: construct the scale and a comparative one of Russian versts. 1 verst = 1167.68 yards.

56. Required, a scale of Belgian fuss $\frac{1}{96}$, and one of Spanish palms corresponding to it.

1 Belgian fuss = .90466 of 1 English foot.
1 Spanish palm = .684 ditto.

57. The plan of a French fort is to be drawn to a scale of 50 English feet to 1 inch, and two other plans of the same fort are to be of the same size; but one reduced to a

scale of metres and decimetres, and the other to a scale of toises and feet—

1 metre = 1.0936 English yards.

1 toise = 76.731 English inches.

58. On a drawing 17.5 feet are represented by a space of 1.3 inches: construct a comparative scale of Spanish pies and pulgados.

1 pie = .9132 of an English foot, 1 pulgado = $\frac{1}{12}$ of a pie or .0761 of an English foot.

59. Reduce 86.72 French metres to English yards.

1 metre = 1.0936 English yards.

60. On a scale 200 yards long, 24 yards = 1 inch, draw a scale of toises comparative to the above, and give its representative fraction. 1 toise = 76.731 inches.

61. Draw a scale of 89.72 yards to $\frac{3}{4}$ of a foot, 100 yards long, and give its representative fraction.

62. Make a scale of Spanish varas and pies showing pulgados diagonally, comparative to another on which 5 yards are represented by 1.89 inches.

1 vara = .9132 of 1 English yard.

1 vara = 3 pies.

1 vara = 36 pulgados.

63. A Prussian fathom contains 6 Rhenish feet, each 1.0297 English feet long: construct a scale of fathoms $\frac{4}{700}$, showing feet diagonally.

64. A map, 40 inches long and 36 inches broad, contains an area of 25 acres: construct its scale to show poles, yards, and feet. 1 acre = 4840 yards.

65. Construct a scale of 13 inches to a mile, showing furlongs and yards.

66. A front of fortification 287.38 yards long is represented by a length of 8.82 inches: show the value of 1 yard and complete the scale to 800.

67. Construct a scale of Bavarian rods and Belgian

ells corresponding to 14.5 English feet to 1.2 inches. The rod = 3.1917 English yards (to show spaces of 10 feet) and the ell = .74845 of an English yard.

68. A range of buildings 2876 feet long is represented by a space of 7.25 inches long: construct to it a scale of 500 yards, showing single yards.

69. Construct a plain scale of yards, 346 yards to 1.7 inch, and also a corresponding scale of Russian sachines. The sachine = 2.3332 yards.

70. On a military plan, 900 feet are represented by a length of 30 inches: construct a comparative scale of yards to it, and give its representative fraction.

71. Two plans on the same ground are drawn, the one to a scale of 2.95 inches to 1 mile, and the other 2.95 miles to one inch: show the length of the line which for each plan will represent 5.57 miles: give also a representative fraction of each of the scales.

DEFINITIONS. (*Euclid*, Book XI.)

12. A pyramid is a solid figure contained by planes that are constituted betwixt one plane and one point above it in which they meet.

13. A prism is a solid figure contained by plane figures, of which two that are opposite are equal, similar, and parallel to one another; and the others parallelograms.

14. A sphere is a solid figure described by the revolution of a semicircle about its diameter, which remains unmoved.

15. The axis of a sphere is the fixed straight line about which the semicircle revolves.

16. The centre of a sphere is the same with that of the semicircle.

17. The diameter of a sphere is any straight line which passes through the centre and is terminated both ways by the superficies of the sphere.

18. A cone is a solid figure described by the revolution of a right angled triangle about one of the sides containing the right angle, which side remains fixed.

If the fixed side be equal to the other side containing the right angle, the cone is called a right angled cone; if it be less than the other side, an obtuse angled; and if greater, an acute angled cone.

19. The axis of a cone is the fixed straight line about which the triangle revolves.

20. The base of a cone is the circle described by that side containing the right angle which revolves.

21. A cylinder is a solid figure described by the revolution of a right angled parallelogram about one of its sides, which remains fixed.

22. The axis of a cylinder is the fixed straight line about which the parallelogram revolves.

23. The bases of a cylinder are the circles described by the two revolving opposite sides of the parallelogram.

24. Similar cones and cylinders are those which have their axes and the diameters of their bases proportionals.

25. A cube is a solid figure contained by six equal squares.

26. A tetrahedron is a solid figure contained by four equal and equilateral triangles.

27. An octahedron is a solid figure contained by eight equal and equilateral triangles.

28. A dodecahedron is a solid figure contained by twelve equal pentagons which are equilateral and equiangular.

29. An icosahedron is a solid figure contained by twenty equal and equilateral triangles.

30. A parallelopiped is a solid figure contained by six quadrilateral figures, whereof every opposite two are parallel.

ORTHOGRAPHIC PROJECTIONS.

The method of orthographic projections enables us to represent, by means of plans and elevations, objects with their exact form and dimensions.

In this method we assume the existence of two planes at right angles to each other, and named from their position, the "horizontal and the vertical planes of projection."

The horizontal, or ground plane, is a plane assumed to be parallel to the horizon. It contains the plans which show the exact form, length and breadth, but *not* the height of the objects to be projected. This plane also often contains auxiliary planes showing the vertical sections when any are required.

The vertical plane is supposed to be standing at right angles with the horizontal plane, and contains the elevations, showing the vertical form, the height and length, or breadth, but not the depth of the objects. On that plane are also usually represented the "sectional elevations."

A section is formed by a plane cutting an object in any direction. When that section is parallel to the horizon, it is termed a horizontal section; when perpendicular to it, a vertical section; when forming any angle with the horizon, an oblique section. A section taken in the direction of its length, a longitudinal section; when taken in that of its breadth, a transverse section; and when taken at right angles with two parallel sides of the object, so as to show the shortest distance between its several parts, a profile section.

As the position of a vertical section can only be determined on a plan by a straight line, in order to show the exact shape and dimensions of the section, we assume an

auxiliary plane revolving on a sectional line from the vertical to the horizontal position.

A sectional elevation shows not only the section just described, but likewise those visible parts of the object which remain after the removal of the nearest portion of the solid determined by the section.

The two planes of projection have often been properly compared to a book half opened, one portion of it lying horizontally and the other vertically. The line formed by the meeting of the leaves would represent the ground line, or line of level, or intersecting line, determining the meeting of the two planes.

In the construction of our diagrams, we suppose the vertical plane to revolve till it lies horizontally on the level and prolongation of the horizontal plane, as if the book just referred to were lying entirely open. The intersecting line is usually determined by two of the last letters of the alphabet, as x and y.

In orthographic constructions.

The plan of an object may be placed at any distance from the vertical plane. The distance between the plan and the vertical plane is termed "region of space." Lines drawn from any point of the plan or elevation, or to represent the position of points in space, are always at right angles with the ground line, and are termed projectors, or system of rays. It follows, therefore, that the projection of a point is determined by the prolongation of a line drawn at right angles from that point till it meets either or both the planes of projection. (Fig. 1.)

Let AB and CD meeting at right angles in x y represent the horizontal and vertical projections of two planes. Let H represent a point in space, a projector Hh drawn parallel to the vertical plane, and perpendicular to the horizontal plane, will not only give its horizontal trace, but will also show its distance from the vertical plane, although not its

height above it. A line Hh' drawn parallel to the horizontal plane, and perpendicular to the vertical plane, will in the same manner determine its elevation or vertical trace. It will show the distance of h' above the horizon, but this will not determine the distance from the vertical plane.

A construction of both traces or projections, however, will give the real position in space and the distance of the given point H from the planes of projection.

Let us now suppose the plane x y CD laid flat, having revolved on $x\,y$ from the vertical to the horizontal position, and in the level prolongation of AB in EF, the projection of h' h'' will determine on that plane precisely the same elevation and position with respect to x y as it does on the plane x y CD.

To determine the projection of a point in space A at .5 inch from the vertical and .5 inch from the horizontal planes. (Fig. 2.)

Draw a a' at right angles to, and cutting x y in a'', make a a'' = to .5 inch, and a'' a = to .5 inch from x y. The traces required a'' a' will determine the height above the horizontal plane, and a'' a will determine the distance from the vertical plane.

The projections of a straight line are determined by an assumed plane (composed of a series of projectors proceeding from every portion of the line at right angles to the co-ordinate planes) connecting the line with the plane or planes of projection, according to its position, and termed projecting plane.

When the straight line is parallel to the horizontal, and at right angles to the vertical planes of projection, its projection will be a line on the horizontal, and a point on the vertical planes.

When the straight line is at right angles to the horizontal and parallel to the vertical planes, its projections

will be a point on the horizontal, and a line on the vertical planes.

The projections of a straight line, parallel to both planes of projection, will be a straight line in each of the planes. And the projections of a straight line, oblique to both planes, will be two unequal lines shorter than the original, and forming angles with $x\ y$, but whose real length can be easily determined.

To determine the true length. Draw $a''\ b'$ parallel to $x\ y$, and at right angles to $a'\ b'$, make $a'\ d$ equal to $a''\ a'$; the line $d\ b'$ will be the true length of the line. (Fig. 8.)

Similarly the projections of a curve, whether regular or irregular, can also be obtained by the projections of points assumed on different parts of the line at right angles to the co-ordinate planes.

Show the projections of a line AB, 1 inch long, parallel to both lines .75 inch above the horizontal, and .5 from the vertical planes. (Fig. 3.)

The lines $a\ b$ and $a'\ b'$, drawn on the co-ordinate planes, parallel and at given distances from $x\ y$, will give the projections.

Show the projection and trace of AB when at right angles to the vertical, and parallel to the horizontal planes. (Fig. 4.)

This is the converse of the preceding operation.

Show the trace and projection of AB when parallel to the vertical, and at right angles to the horizontal. (Fig. 5.)

The horizontal will be the point $a\ b$, and the vertical projection the line $a'\ b'$.

Determine the projection of AB when horizontal, but forming an angle of 30° with the vertical plane.

$a\ b$ drawn at an angle of 30° with $x\ y$, and $a'\ b'$ drawn parallel to $x\ y$, will give the projections. (Fig. 6.)

Let the line AB form an angle of 30° with the horizontal, and be parallel to the vertical. (Fig. 7.)

Determine the projection *a′ b′*, from which find *a b*.

Let the line be inclined to both planes of projection at angles of 30°.

The two last operations repeated on both planes will give the projections of *a b* and *a′ b′*. (Fig. 8.)

As the boundaries of planes are lines, the projections of planes will easily be determined according to their position.

To determine the projection of a rectangular plane ABCD of .75 inches side, parallel to the vertical plane, and .5 inch distance from it. (Fig. 9.)

The rectangle *a b c d* determine the vertical, and the line *a′ d* the horizontal projections required.

Required the horizontal and vertical projection of a circular plane 2 inches in diameter, parallel to and 1 inch above the horizontal plane.

The mere inspection of the diagram should now suffice for the solution of this question. (Fig. 10.)

Show the projection of a pentagonal plane of .5 inch side. One of whose sides forms an angle of 15° with the vertical plane, but parallel and 1 inch above the horizon. (Fig. 11.)

Draw *y x b*, forming an angle of 15°, and on *a b* .5 inch side construct the pentagon *a b c d e*. Its height above the horizon being given, determine its elevation by means of the projectors, *a a′ b b′ c c′ d d′ e e′*.

Solids are obtained in the same manner.

A cube of .75 inch edge has one of its faces parallel to the vertical plane, find its projections.

Construct the plan *a b c d*, and from it determine its elevation. (Fig. 12.)

A pentagonal prism .75 inch high, has one of its faces

.65 inch side, forming an angle of 21° with the vertical plane: construct its projection.

Determine the angle by making AC at 21° with $x\,y$, and make BC equal to .65 inch. On BC construct the regular pentagon, forming the plan of the solid: lines drawn at right angles from each angle of the base, through $x\,y$ and .75 inch high, connected at their superior extremity by a horizontal will give the elevation. (Fig. 13.)

A pentagonal pyramid, .65 inch side and 1.1 inches high, has one of its faces forming an angle of 10° with the vertical plane, construct its projections. The construction of this problem is similar to the preceding, the only difference is that the height is determined on $a\,b$, the axis of the pyramid. (Fig. 14.)

The two parallel sides of a square truncated pyramid .75 inch high, are respectively .75 and .33 inch side, one of the sides forms an angle of 40° with the vertical plane: construct its projections. (Fig. 15.)

A parallelopiped 1 inch long, .65 inch side and .65 inch high, has its longest side parallel to the vertical plane: construct its projections. (Fig. 16.)

Construct the projections of a cylinder .85 inch high and .65 inch diameter. (Fig. 17.)

Show the projections of a cone 1 inch high, and whose base has a diameter of .8 inch. (Fig. 18.)

The frustum of a cone .7 inch high, and .8 inch diameter at the base, has its upper face .35 inch broad: show its projections. (Fig. 19.)

The construction of the five last problems being, with slight variations, mere repetitions of those already explained, need no further demonstration; the inspection of the diagrams ought to suffice.

Regular Polyhedrons.

A tetrahedron of 1.5 inches edge has one of its sides forming an angle of 17° with the vertical plane: construct its projections.

Make the angle $b\,x\,y$ equal to 17°, and $a\,b = 1.5$ inches; on $a\,b$ construct an equilateral triangle $a\,b\,c$ for the base, and complete the plan by drawing lines from each of the angles, meeting in the centre d. Draw $d\,e$ perpendicular to $a\,d$, and produce $a\,d$ to f. From centre f, with radius $f\,a$, describe the arc $a\,e$; $d\,e$ will determine the height of the tetrahedron $d'\,d''$, which is to be completed as usual. (Fig. 20.)

The hexahedron, or cube, in its different positions will be explained further on.

An octahedron of 1.25 inches edge is laid on one of its faces, whose edge forms an angle of 15°, with the vertical plane: construct its projections.

Let $a\,b$ 1.25 inches form the side or edge of one of the faces of the octahedron at 15° with $x\,y$. On $a\,b$ construct the equilateral triangle $a\,b\,c$ from g as a centre, (which is to be determined); describe a circle passing through $a\,b\,c$, and in the circle inscribe the hexagon $a\,d\,b\,e\,c\,f$, and construct the triangle $d\,e\,f$; draw the diameter $c\,d$, and erect $d\,k$ perpendicular to $d\,c$.

From l as a centre, with radius $l\,c$ describe the arc $c\,k$.

$d\,k$ will be the height of the octahedron, which will determine that of the elevation.

The inspection of the diagram will guide in the completion of the problem. (Fig. 21.)

A dodecahedron of .65 inch side has one of its edges forming an angle of 32°; with the vertical plane construct its projections.

Determine the angle $b\,x\,y$, and on the given side $a\,b$ construct the equilateral and equiangular pentagon $a\ b\ c\ d\ e$, forming the base of the solid. Circumscribe the pentagon by a circle passing through its angles, and bisect each of its sides by a perpendicular produced on both sides, and bisecting each of the opposite angles. The five intersecting points, $f\,g\ h\ k\ l$, thus determined on the circumference, will determine the angles of a second regular pentagon for the upper face of the solid.

Make $n\ o$ equal to $m\ n$, and from m as a centre, with radius $m\ o$; describe a circle $o\ p\ q\ r\ s\ t\ u\ v\ w\ z$, which will contain a regular decagon formed by the intersection of the lines bisecting the sides and angles of the lower and upper faces, and produced so as to meet its circumference. Complete the plan as shown in the diagram. (Fig. 22.) For the elevation, from each angle of the plan draw perpendiculars produced through $x\ y$; make $a'\ p'$ equal to $m\ a$ and draw $r'\ p'\ z'$ parallel to $x\ y$; make $p'\ u'$ equal to $a\ p$, and draw $s'\ u'\ w'$ parallel to $x\ y$; make $u'\ k'$ equal to $a'\ p'$, and draw $l'\ h'$ parallel to $x\ y$: these horizontals will determine the heights in the elevation, which is to be completed as shown in the diagram.

An icosahedron of .85 inch edge is laid on one of its faces, and has one of its sides forming an angle of 30° with the vertical plane.

Determine the angle $a\ y\ x$, and on the given side $a\ b$ construct the equilateral triangle $a\ b\ c$, forming the lower face of the solid; determine its centre d, and circumscribe it by a circle containing the second equilateral triangle $e\,f\,g$ (the upper face of the solid). On $b\ c$ construct the regular pentagon $b\ c\ h\ k\ l$, bisecting each of its angles by lines meeting at its centre m. With radius $d\ m$ describe a circle containing the hexagon $m\ n\ o\ p\ q\ r$, and complete the plan as shown in the diagram. (Fig. 23.)

The elevation is obtained by drawing as usual the projectors perpendicular and produced through $x\ y$; from b' as a centre describe the arc k', cutting the projection of f in f', and join $b'\ f'$.

From b', with radius $b\ s$, describe the arc cutting $b'\ f'$ in s'; through s' draw $p'\ m'$ perpendicular to $b'\ f'$, and draw $f'\ e'$ parallel to $a'\ b'$; draw likewise $a'\ e'$ parallel to $b'\ f'$; join $e'\ m'$, $b'\ p'$ and $g'\ r'$: complete the solid as shown in the diagram.

A cube 1 inch edge has one of its faces forming an angle of 22° with the vertical plane. Construct its projections, also a section cutting two adjacent faces of the solid, and a sectional elevation. (Fig. 24.)

After having determined the angle, the plan A and elevation B are constructed in the usual manner.

Draw anywhere the sectional line $c\ d$, and draw $c\ f$ and $d\ e$ at right angles to $c\ d$, and equal to the height of the cube, and join $c\ f\ e\ d$, which will show the section required—as if supposed to have revolved on $c\ d$ from the vertical to the horizontal position.

For the sectional elevation C make the parallelogram $c\ d\ e\ f$ equal to the section $c'd'e'f'$, and draw GK, HL, at right angles to $c\ d$, produced to k and l: make $k'\ c'$ equal to $k\ c$, and $d'\ l'$ equal to $d\ l$, and erect the perpendiculars $k\ m$ and $l\ n$ equal to the height of the solid; draw $m\ f'\ e'\ n$ parallel to $x\ y$, completing the sectional elevation.

Whenever no conditions of light and shade are given, it is usual to suppose the light to fall from the top left of the picture, at an angle of 45° with the vertical plane, and 45° with the horizon. Those planes or parts of the diagrams supposed to be exposed to the light are drawn in thin lines, and those opposed to the light, and consequently in the shade, are inked in with thicker lines.

Lines determining portions of the object unseen in the

diagram, and yet which are necessary for its construction, are dotted.

Sections are usually distinguished by being drawn with red ink, and the projections of shadows with blue ink: in the accompanying diagrams, however, the sections will be represented by thick bars, and the shadows by thick chain lines.

In sectional elevations, in order to distinguish sectional from original planes, it is customary to fill them up with oblique parallel and equidistant lines, whose space from each other vary with the size of the drawing.

A tetrahedron of 1.5 inch edge has one of its sides forming an angle of 17° with the vertical plane: show its plan, elevation, section, and sectional elevation—the section to cut three faces of the solid. (Fig. 25.)

The construction of the tetrahedron has already been explained.

Let *a b* represent the sectional line, *c* and *d* the points through which the sectional plane cuts the solid; from *c* and *d* draw perpendiculars to the elevation, cutting the projectors of these edges in *c′* and *d′*; these points will determine the heights in which those edges are cut above the base; from *c* and *d* on the plan, and perpendicular to *a b*, draw *c c′ d d′*, equal to their corresponding heights on the elevation; lines connecting these points and drawn to *a b* will give the section.

For the sectional elevation (Fig. 25 *a*), reproduce exactly the section *a c″ d″ b*, from the apex *f* draw *f g* perpendicular to the sectional line *a b*, and transfer the distance *a g* from the section to *a′ g′* on the sectional elevation; make *g′ h* equal to the height of the tetrahedron, and complete the figure as in the diagram.

Show the plan of a parallelopiped 2 inches long, 1 inch broad, and 1.25 inches high; show also a section on a

line cutting an angle of the solid to the bisection of one of its shortest sides.

The mere inspection of the diagram should demonstrate its construction.

A right pyramid, 1.8 inches high, has one of the sides of its base, which forms a regular pentagon of 1.1 inch side, making an angle of 26° with the vertical plane. Construct its projection, also a transverse section passing through its axis perpendicular to the vertical plane, and inclined to the horizon at an angle of 31°. (Fig. 27.)

Construct the plan and elevation of the solid as in the preceding examples, and on the elevation determine the section line *s l* according to conditions, and cutting the edges of the pyramid in a' b' c' d' and e'; from these points drop perpendiculars so as to intersect on the plan the corresponding edges of the solid in *a b c d* and *e*: join these points, and the sectional plan of the pyramid will be produced.

To show the real section; on the plan, draw *a f* parallel to *x y*, and from 1, 2, 3 and 4 draw ordinates perpendicular to it, meeting the angles of the section; at b' c' d' and e' draw likewise perpendiculars $1'$ $2'$ $3'$ and $4'$ to a' e', and transfer on these the length of the ordinates on the plan; connect *a* 1 3 4 2 and a', which will give the real section reduced to the vertical plane.

A pentagonal pyramid 1 inch side and 1.75 inches high has one of the edges of its base forming an angle of 24° with the vertical plane: construct its projections, and also a section cutting 4 of its sides perpendicularly to the horizon, but not passing through the apex. (Fig. 28.)

After having obtained the plan and elevation in the usual manner, repeat the operations already explained (Fig. 25) to obtain the section, viz.—

Draw the sectional line $s\,l$ intersecting 4 sides and 3 edges.

Draw $a\,a'$ $b\,b'$ $c\,c'$ perpendicular to $x\,y$, and draw $a\,a''$ $b\,b''$ $c\,c''$ perpendicular to $s\,l$, and of the same altitude as their corresponding heights on the elevation; join s a'' b'' c'' and l for the section required.

The inspection of the two following diagrams should be sufficient for their apprehension.

The profile section of a right prism is an equilateral triangle of 1 inch side, the prism is 1.75 inches long and rests on one of its faces: construct its plan and elevation, showing one face and one end of the solid; also a section on any plane cutting both faces and both ends. (Fig. 29.)

A prism 1.75 inches long, having for its ends regular hexagons of .5 inch side is laid on one of its faces: show its plan, elevation, and a section taken on a line cutting all the sides and one end of the solid. (Fig. 30.)

OBSERVATION.—Before going on with the following problems, the general student, the military candidate whose time for preparation is not too closely limited, is referred to the article on "Descriptive Geometry" (page 112), the proper acquisition of whose elements is recommended as a safe, although slower foundation for his future progress: the following problems being mostly practical applications based on questions given at the examinations.

A circle of 1.5 inches diameter is placed on a plane forming an angle of 46° with the horizon. (Fig. 31.)

(N.B.—The intersections of a plane with the two intersecting planes are termed 'the traces of the plane,' and a plane is determined when the traces of that plane are given.)

Let $a\,b$ represent the horizontal trace acting as an axis of revolution to the plane $b\,c$, so as to determine its given inclination with the horizon upon the vertical plane.

Let *d e f g* represent the two diameters and *k* the centre of the given circle; draw the perpendiculars *d d′*, *g k′* and *e e′* meeting *x y*, and from *b* as a centre describe the arcs *d′ d″*, *k′ k″* and *e′ e″*. Lines drawn perpendicularly to *x y* from *d″* to *n*, *k″* to *m* and *e″* to *o*, will determine the projections of the two diameters of the circle, which, in their new position, will form the major axis *l m* and the minor axis *n o* of an ellipse, which is to be completed by drawing the curve *l n m o* by the hand.

It is evident from the foregoing problem that the oblique projection of a circle is an ellipse. The solution of the next exercise will therefore be easy, and require no further explanation.

The side of a cylinder 1.75 inches long and 1 inch diameter, forms an angle of 23° with the vertical plane. Construct its projections. (Fig. 32.)

A plane forming an angle of 40° with the horizon, contains on its surface a line 1.25 inches long, inclined to the horizon at an angle of 30°. Contrast its projections. (Fig. 33.)

A plane can contain a line having an inclination less or equal to its own, but never a line having an inclination greater than that of the plane, for in order to be contained by a plane the two extremities of the line must coincide with it. And should the inclination of the line be greater than that of the plane, the line could only touch the plane at one point.

Draw the traces *a b* and *b c*, forming the given angle (40°) with the horizon.

Draw anywhere the line *d e* 1.25 inches long, forming with *x y* the given angle (30°.) Draw *e f* parallel to *x y*, *f* will be the height above the horizon of the line *d e* when received by the given plane. From centre *b*, with radius

$b\,f$ describe the arc $f'\ f''$, and draw $f''\ k$ parallel to $a\ b$. The distance $b\ f$ being equal to the distance $b\ f''$, the inclined plane $a\ b\ f'$ will evidently be reduced to the horizon in $f''\ k$: take in the compass the distance $d\ e$, and transfer it, so that the two extremities may coincide any where on the two horizontals $a\ b$ and $f''\ k$ in g and f. Draw $f\ l$ parallel to $x\ y$ and $f'\ l$ parallel to $a\ b$, and intersecting it, $g\ l$ will be the horizontal projection of the given line.

A square 1.25 inches side is placed on a plane forming an angle of 30° with the horizon, and one of its sides is inclined at an angle of 15°. Show its plan. (Fig. 34.)

The first part of this exercise (excepting the difference in the value of the anglès) is a mere repetition of the last problem, and, therefore, requires no fresh demonstration.

On $g\ f$ complete the square $m\ n$, and draw $m\ \ o$ and $n\ p$ perpendicular to $x\ y$, from centre b describe the arcs $o\ m'$ and $p\ n'$, and draw $m'\ \ m''$ and $n'\ n''$, meeting the lines $m\ m''\ \ n\ n''$, drawn parallel to $x\ y$.

The side of a pentagonal prism 1 inch long and .35 inch high is inclined at an angle of 20° with the horizon, and stands on a plane, forming an angle of 50° with the horizon. (Fig. 35.)

No matter what may be the shape of the figure, the line first obtained will ever serve as a base of construction to it. On the line $g\ f$, therefore, construct the pentagon and project it as before. On $b\ c$ make $b\ d$ equal to the given height (.35) and lines drawn parallel to it in $h\ \ l\ \ m$, and also to $b\ c$ in d and e will lead to the completion of the horizontal projection of the solid.

To obtain the horizontal projection of a regular solid it is not always necessary to complete its vertical projection. The position and heights of its different parts being given will be sufficient, as

A heptagonal prism .25 inch high and 75 inch side has

one of its sides forming an angle of 20°, and is placed on a plane inclined at an angle of 60° with the horizon. The prism is surmounted by a heptagonal pyramid 1 inch high, whose base coincides exactly with the upper surface of the prism. Show its horizontal projection.

The heights *a b* .25 and *c d* 1.25 showing the position and combined height of the prism, and of the axis of the pyramid will suffice, as the inspection of the diagram will show; to determine the horizontal projection required. (Fig. 36.)

Two sides of a triangle, respectively .75 and 1 inch long, meet on the horizontal plane, the first has its extremity at .25, and the second at .5 inch above the horizon. They are contained by a plane inclined to the horizon at an angle of 50°. Complete the triangle, and determine the real angle contained by the two lines as well as their horizontal projection. (Fig. 37.)

Construct the angle *a b c*, and draw any where the perpendicular *d e* equal to .25 inch, and *d f* equal to .5 inch. Make *f h* equal to .75 inch and *e g* equal to 1 inch. The plane *a b c* being reduced to the horizon will contain the two lines *f' h'* and *f' g*, equal to *f h* and *e g*, the real angle between which has to be measured, and the horizontal projection of the triangle to be completed as usual.

A plane forming an angle of 42°, contains an isosceles triangle of 1.5 inches side, whose base of 1 inch is inclined to the horizon at an angle of 29°, show its projection. (Fig. 38.)

This exercise requires no explanation, being a mere repetition.

When the inclination of two sides of an object are given, to determine its horizontal projection and the inclination of the plane containing it, as,

Two adjacent sides of a square 1.25 inches side are respectively inclined to the horizon at angles of 34° and 20°.

Construct the plan of the square, and determine also the inclination of the plane on which it is situated.

Draw *a b, a c,* the two sides of the square, each 1.5 inch long, and forming at *a* an angle of 90°. Make the angle *a b d* equal to 34°, and the angle *a c e* equal to 20°. Draw *a d* at right angles to *b d,* and from *a* as a centre, with radius *a d* describe the arc *d f,* draw *a f* through *e* at right angles to *e c,* and draw *f g* parallel to *e c,* and meeting *a c* produced in *g.* Draw the axis *g b,* produced towards *k.* This axis *g b* will be a horizontal or line of level on which the plane containing the figure is supposed to revolve. Complete the square by drawing *c h* and *b h* respectively at right angles to *a c* and *a b,* and construct the elevation by drawing *x y* at right angles to *g b k.* Make *k m* equal to *a d,* and draw *k n* parallel to *x y.* Draw the projectors *a*-1, *c*-2, and H-3 to *x y,* and parallel to *g k.* From centre *m,* with radius *m*-1, describe the arc 1-0, meeting *k n* in *o,* and from the same centre describe also the arcs 2-*p* and 3-*q,* meeting the line *o q* drawn through *m,* and determining likewise the real inclination of the plane containing the square, the lines drawn from *o, p* and *q,* parallel to *g k,* meeting lines drawn parallel to *x y,* in *a a′, c c′,* and *h h′,* will determine the position of the corners of the square. (Fig. 39.)

Find the value of the angle *x m o.*

The following construction is a mere repetition of what has just been explained. The two adjacent sides *a b* and *a c* of a parallelogram are respectively 1 and 1.65 inches long, *a b* is inclined to the horizontal an angle of 38°, and *a c* at an angle of 16°. Find its projection and determine the inclination of the plane on which it is situated. (Fig. 40.)

A cube of 1 inch edge has two of its sides forming angles of 50° and 25° with the horizon, its base coincides with

that of a right square pyramid 3 inches high, showing on its surface at $\frac{2}{3}$ of its height lines parallel to its base—determine the projections of the cube and pyramid, showing also the intersection of the two solids.

It is evident that the first part of this problem is in every respect similar to those already described. On the line of inclination, *o q*, construct the elevation of the cube *o r*, *p s*, *m t*, and *q u*, and determine likewise the height of the axis of the pyramid *v w* at $\frac{2}{3}$ of which at *l* draw a line at right angles to it, and determine the horizontal projection in the usual manner. (Fig. 41.)

Two sides 1.25 inches long of a triangular right pyramid 1.5 inches high, are respectively inclined at angles of 52° and 37° to the horizon, determine its projections. (Fig. 42.)

The preceding problems being understood, no difficulty whatever should be experienced in the construction of this and of the four following questions, which should require no further explanation, and be understood by mere inspection.

A triangular prism ·1 inch side and .25 inch side, has two of the edges of its faces, forming respectively angles of 40° and 55° to the horizon, construct its projection. (Fig. 43.)

A pentagonal pyramid, 1 inch side, 2 inches high, has two of its edges inclined respectively at angles of 23 and 14° respectively to the horizon. Show its projections, and determine on its surface a horizontal, determined at $\frac{1}{3}$ its height taken on its axis. (Fig. 44.)

A block 1 inch long, .75 inch broad, and .5 inch high, contains on its upper surface an ellipse whose major axis 1 inch long is inclined to the horizon at an angle of 50°, and its minor axis, .75 inch long, forms with the horizon an angle of 25°, construct its projections. (Fig. 45.)

A square block .8 inch side and .15 inch high, is surmounted by a cylinder .6 inch high, terminated by a cone

.65 inch high, whose base coincides with the top of the cylinder and whose surface is tangent to the sides of the block. Construct its projections. (Fig. 46.)

A plane inclined at an angle of 25° to the horizon has its horizontal forming an angle of 40° with the vertical plane. Construct its projections. (Fig. 47.)

On *x y* make the angle ***a b c*** equal to 40°: ***b c*** will be the horizontal required. Draw ***a c d*** at right angles to ***c b***, and make the angle ***a c d*** equal to 25° (the angle which the plane makes with the horizon). Make ***a d*** at right angles to ***a c***, and ***a e*** equal to ***a a*** at right angles to ***a b***. Join ***e b***.

Application.

The projections of a cube .85 inch edge standing on a plane inclined to the horizon at an angle of 25°, and whose horizontal trace makes an angle of 40° with the vertical plane. One edge of the cube making with this trace an angle of 50°: let the lowest face be considered likewise as the base of a right pyramid 1.65 inch high. (Fig. 48.)

The projections of the plane are determined in the manner that has just been explained.

Draw the side of the square CH at the angle of 50° with the horizontal trace *b c*, and project it on the elevation *c a*; reduce it now to its horizontal projection HC, and complete the plan of the square H C K L, and through the elevation on *c d*. Complete its projection in K′ L′, complete likewise on *c d* the profile elevation of the cube in *m p*, *n q*, *o r*, and *c s*, and construct its horizontal projection on G H′ K′ L′ (the edges being at right angles to *c b*.) On *c d* determine the profile elevation of the square pyramid *m n o c*, having its apex in *t*, its horizontal projection will be determined by lines parallel to *c b*. To construct the elevation on the vertical plane, parallel to *c a*, draw the lines *o o′*, *n n′*, *m m′*, and from *a* as a

centre describe the arcs *m m''*, *n n''*, *o o''*, and from *m''*, *n''*, and *o''* draw lines parallel to *a b*. From every point in the figure horizontally projected, erect perpendiculars, cutting those lines at the intersection of which other lines drawn perpendicular to *b e* will determine the position of the lines required to complete the elevation.

I have endeavoured thus far to reduce to certain fixed rules the method of Orthographic projections, and by constructing every given subject in the same manner, and according to the same principles, my aim was to demonstrate the analogy between each. There is nevertheless no fixed and universal method of projections, the results and principles are the same, but the methods vary. The diagrams 49 and 50 are examples to the purpose.

A cube .75 inch edge has one of its faces inclined to the horizon at an angle of 30°, construct its projections.

Draw *a b* perpendicular to *x y* for an auxiliary plane, and construct the plane A, according to dimensions.

Construct the plane B at 30° for the inclination of the base of the solid. (Fig. 49.)

Horizontal and perpendicular projectors, as seen in the diagram, will first give the elevation B, and then enable the elevation C and horizontal projection D to be constructed.

A cylinder 1.25 inches diameter and 1.5 inches high, has its axis inclined to the horizon at an angle of 28°, construct its projections. (Fig. 50.)

Let the circle A B C D represent the plan of the cylinder. On AC as a base draw AE at the given angle (28°.) On AE as a base draw the elevation A G E F; the perpendicular FH, GK, will determine the minor axis of the upper elliptical surface, and LA that of the under face. The projection of the diameter BD, parallel to CK, will determine the breadth of the major axis: inspection will show the rest.

In mechanical and architectural drawing it is often necessary to represent the intersection of solids, or surfaces of different kinds, such as the junction of cylinders, pipes of different diameters, and at different angles, the crossing of arches, bridges, &c. These cases are solved in the following manner:—

Two cylinders of equal diameters intersect in such a manner that their axes cross at right angles to each other. (Fig. 51.)

Inspection of the diagram will show that the intersection of cylinders of equal dimensions, and under these conditions will form in plan the cross *a b c d*.

The axes of two cylinders of different diameters, intersect at right angles; show their projection. (Fig. 52.)

Let *a* and *b* represent the vertical and horizontal projections of the cylinders.

Produce the points *c e* in *c e′ c e″*, and draw the semicircle GGG; and the points *f f* in *f′ f″*; draw any where the horizontal *d d* parallel to *c c*, and cutting the circle in *e e*, and the semicircle in KH. Draw *e e′ e e″* parallel to *c e′* and *c e″*, take in the compass the distance HK, and transfer it from the axis C′ C″ right and left to LL on the lines *e′ e″*.

A curve drawn through *f l c′ l′* will give the horizontal projection of one side of the intersection. The other side is to be completed in a similar manner.

The axis of two cylinders of different diameters intersect obliquely, show their projection. (Fig. 53.)

The inspection of the diagram will show that the principles of construction are precisely similar to the above.

Application.

The soffit, or intrados of an arched passage 32 feet wide, is intersected at right angles by an arched passage

of 16 feet wide, in such a manner that the axes of their semicircular arches are also intersecting. Show the plan of the intersection of the arches on a scale of 10 feet to 1 inch. (Fig. 56.)

The inspection of the first part of the preceding question will give the solution of the present case.

Two cylinders of different diameters laid on their side intersect in such a manner that their lower surfaces coincide.

First case, fig. 54, *a* and *b* shows the elevation and plan of the cylinders when at right angles to each other. Inspection of the diagram will show the analogy between this construction and that of diagram 52.

The points of intersection of the cylinders, whatever may be their position, are determined by a series of horizontals taken at will on the elevation, and determined on the plan by a series of lines at right angles to them. Fig. 55 is a further demonstration of the same problem applied to cylinders oblique to each other, but following the same principles of construction.

Determine the penetration of a sphere by a right cone. (Fig. 57.)

Draw the horizontal and vertical projection of the solids in the usual manner.

The projectors $a\ a'$, $b\ b'$, $c\ c'$, will determine the lower curve of penetration, and $d\ d'$, $h\ h'$, and $f\ f'$ will determine the upper one.

From the centre A with radius AC describe the arc $g\ a\ g$, meeting the horizontal projections of the generatrix of the cone, the curve $g\ a\ g\ b$ will show their intersection.

From the centre A with radius Ah describe in a similar manner the curve $h\ e\ h$, and describe the curve $h\ d\ h\ f$.

Two right cones, with bases of different diameters, in-

tersect with their axis oblique to each other. Show their curves of penetration.

Fig. 58, *a* and *b*, show the plan and elevation of the solids. On the elevation, and at right angles to the axis, *c d*, through the points of intersection; draw the horizontals *c* 1, *f* 2, 3 *h*, and 4 *k*.

From centre A in plan, with radius I *e*, 2 *f*, 3 *h*, and 4 K, describe arcs, if the intersecting cone is oblique to the vertical plane, determine the distances from *a* to *e'*, *f'*, *h'* and *k'*, if the cone is parallel to it. The inspection of the diagram will show how to complete the figure.

Show the intersection of a right cylinder with a cone when their axes are at right angles with each other.

Should the axis of the cylinder be oblique to the cone, the mode of its construction would but slightly vary with that just explained. And the inspection of the diagram 59 will show that the present question is based on precisely the same principles, and that its solution is equally simple.

The five horizontals, 1, 2, 3, 4, 5, through the elevation will determine the diameters of the curves as 1′ 2′ 3′ 4′ and 5′ in plan, and show the points of intersection of the solids, (*vide* diagram.)

The intersection of a sphere by a cylinder also follows similar rules. (Fig. 60.)

Construct the plan and elevation of the sphere, and determine the position of the plan of the cylinder, through which draw the lines 1, 2, 3, 4, parallel to *x y*, through the points of intersection of these lines with the circumference of the cylinder, draw verticals produced through its elevation.

From E as centre, with radii 1 *a*, 2 *b*, 3 *c*, and 4 determine the points *a′*, *b′*, *c′*, and *d′* on those perpendiculars, and complete the curve with the hand.

Two right cones having bases of different diameters intersect, show their projection. (Fig. 61.)

Let *a b* equal the diameter of the larger cone at its base, *c* its apex.

Let *d e* equal the diameter and position of the smaller cone, and *f* represent its apex.

Construct their elevations in *a′ b′ c′*, and in *d′ e′ f′*. Through *f*, *c*, draw *h k*, at right angles to which draw the profile section of the cones *h f′′ g* and *h c′′ k*, intersecting in *y′*. With radius *c y* describe the circle *l l*, and project it in *l′ l′*; draw *h h′* perpendicular to *x y*, and join *h′* and *c′*, intersecting *l′ l′* in *m*. From the centre *f*, with radius *f y*, describe the circle *y z*. (The circles *l l* and *y z* will be horizontals of the highest point of intersection of the cones), bisect *y h* in *o*, and describe the circle *y h* for the horizontal projection of the elliptical section of the two cones. From *c* describe any circle *p s*, cutting *y h* in *r r*, and project it in *p′ p′*, draw *r r* parallel to *a′ b′*, cutting *p′ p′* in *r′ r′* : *r′ r′* will become two points, through which the sectional ellipse will pass: draw *t t′* tangent to the circle *y r h r*, and cutting *f′ e′* in *t′*. Through the points *h′ r′ r′ t′* and *m* draw the ellipse.

Required the projections of a cylinder of 1 inch diameter, and 1.5 inch high, lying on its side perpendicular to the vertical plane. Show the section that would be produced by a plane passing through and making an angle of 45° with its axis. (Fig. 62.)

Divide the circumference of the circle forming the elevation into any number of equal parts, 1, 2, 3, 4, 5, 6, &c.

Below *x y*, draw the plan according to conditions of position, breadth, height, &c.; from each point on the elevation drop perpendiculars 1′1′, 2 2′′, 3′3′, 4′4′, 5′5′ and 6′6′; and for the section draw *f g* at the given angle with the axis, and draw perpendiculars to it through each point

where it is crossed by the lines parallel to the axis. Make these perpendiculars equal in length to their corresponding lines in the elevation.

APPLICATION.

Draw the plan of a length of 30 feet of a semicircular arch of 18 feet span and 6 feet thick, standing on abutments or side walls 10 feet thick, and 10 feet high to the springing of the arch. Draw a section through the arch and abutments on a line inclined to the axis of the arch at an angle of 43°. Scale 20 feet to 1 inch. (Fig. 63.)

The preceding question being understood, this requires no further explanation.

Show an inner arc of 130° of a circular tunnel, semicircularly vaulted, whose span equals 10 feet, and one of the extremities of which is at right angles with its axis. The other forms with it an angle of 40°, show the two sections. Also a third vertical section on a segment of the arch, in any direction, but passing through the crown of the arch. (Fig. 64.) Scale 10 ft. to 1 inch.

Like No. 63, this figure is evidently an application of fig. 62, and therefore needs no further explanation.

Show the projection of a right cylinder standing on the plane of its base, having a rope twisted spirally round it, and having an inclination or pitch of 1 in 3. (Fig. 65.)

Like in the preceding problems, divide the circumference of the circle forming the base into any number of equal parts or sections (say 12), for the semi-circumference. For the elevation, construct on $x\ y$ the parallelogram $a\ b,\ c\ d$, equal to the height required. From each of the sectional points on the plan meeting the semi-circumference draw lines 1-1′, 2-2′, 3-3′, 4-4′, 5-5′, 6-6′, 7-7′, 8-8′, 9-9′, 10-10′, 11-11′, through $x\ y$ and parallel to the axis of the cylinder. To obtain the pitch, say 1 in 3, take in your

compass that fraction ($\frac{1}{3}$) of the arc of one of the sectors (say 1-2), and with that distance draw on the elevation a series of equi-distant horizontal and parallel lines, whose intersections with the verticals corresponding to the sectorial points, will determine the curve which is 'to pass through them.

N.B.—As this problem is applied to the construction of circular staircases, helices, screws, and curves of regular pitch, &c., the principles of which are precisely the same, the five following problems will only require slight modifications for their solution.

The two diameters and pitch of a square screw being given, construct it upon the plan.

Draw the two concentric semicircles *a b, c d,* equal to the given diameters, and from the central one erect a perpendicular for the axis of the screw. Construct the elevation, and divide the circle explained into the number of required parts. The inspection of the diagram will show that the problem is to be completed in the same manner as the one just explained. (Fig. 66.)

The two diameters and pitch, say $\frac{1}{12}$ of a V screw being given, construct it. (Fig. 67.)

The two diameters and pitch of a spiral staircase being given, show its projections. (Fig. 68.)

These two diagrams need no further explanation, for their inspection will show the analogy of their construction with the problems already given.

Around the surface of a cone construct a spiral of regular pitch. (Fig. 69.)

Divide the plan into any convenient number of equal sectors, and on each point determined on the circumference draw perpendiculars to *x y,* meeting the base of the elevation. Divide the altitude of the cone into a series of horizontal contours proportioned to the rate of inclina-

tion required for the pitch, and from every point at the base of the elevation draw lines *a b c d e*, converging to the apex. The curve drawn through the points of intersection of the radiating lines, with the contours, will give the elevation of the curve, and perpendiculars projected downwards through *x y*, to their corresponding horizontal projection will give the plan of the curve.

Around a hemispherical surface, or a sphere, to construct a spiral, having a given regular pitch. (Fig. 70.)

The only difference from the preceding construction is in the elevation. The ribs being converted into vertical arcs, and obtained as in the construction of arcs of longitude in geographical charts. A line bisecting the distance *a k* will meet *x y* in 1, which will become the centre of the arc *a k*; in the same manner the arc *b k* will have its centre at 2, *c k* at 3, *d k* at 4, *a k* at 5. These centres transferred on the opposite side of the axis *k k* will likewise give the position of the other arcs.

The remaining part of the figure is obtained as already explained.

Show the projections of a right cone 1.5 inches high, standing on the plane of its base, which is 1 inch in diameter. Show the position of a point situated on its surface at a perpendicular height of 1 inch above the base, appearing at .25 inch from its right boundary line, as seen in its elevation.

Draw $a' b'$ parallel to *x y*, and 1 inch from it: on it determine at .25 from b' the point c', through which draw $e' c' d'$. From centre *c* describe the arc *a b*, horizontal projection of $a' b'$, draw $d' d$ perpendicular to *x y*, and join *d e*, intersecting the arc *a b* in *c*, the horizontal projection of the point required. (Fig. 71.)

Given the horizontal and vertical projections of a cone. Show an elliptical section, the major axis forming a given angle with the horizon.

Divide the height $a\,a'$ into any number of horizontal, equidistant, and parallel contours. Divide the semicircular base $c\,d$ into the same number of concentric circles corresponding with these contours. Let $f'\,g'$ represent the inclination of the sectional plane. From the points F′ 1′ 2′ 3′ and g' draw perpendiculars cutting their corresponding contours in f, 1, 2, 3, and g. From 1′ 2′ 3′ draw perpendiculars to $f\,g$ equal in length to the distance of the points 1′ 2′ 3′ from the base line $c\,d$. Through those points draw the curve. (Fig. 72.)

Application.

Draw the plan of a conical mound of earth 300 feet in diameter and 80 feet high, on a scale of 100 feet to 1 inch. Show on its plan, contours at 10 feet vertical intervals, and draw the horizontal projection of the surface that would be exposed if this mound were cut by a plane inclined to the horizon at an angle of 15°, and intersecting the mound at a height of 70 feet.

Determine the projections of the parabolic section of a given cone.

A few moments' consideration will show the analogy between this and the preceding figure. The construction, being based on precisely the same principles, requires no further directions. (Fig. 73.)

Application.

A cone 3 inches high, standing on a base described with a radius of 1.25 inches, is cut by a plano parallel to its generatrix, and at a distance of 1 inch from it: draw its plan, and the intersection of the plane with the cone, and show also the actual outline of the section formed by the plane with its surface.

Show a given hyperbolic section of a cone. As the hyperbolic section of the cone would only be represented

in plan by a straight line $f\,g$, its elevation would be determined by verticals drawn from the intersection of that line with the contours and with their corresponding horizontals in the elevation. The hyperbola, therefore, would be determined by perpendiculars taken on $f\,g$ at the points 1 2 3 4 5 6 7 8, and cutting the horizontals in 1′ 2′ 3′ 4′ 5′ 6′ 7′ 8′, &c. (Fig. 74.)

Application.

Two right cones, 3 inches high and 4 inches in diameter at their base, whose axes are parallel and at 1 inch distance from each other, intersect: draw the section produced by that intersection.

Developments.

The development of a solid is a Geometrical mode of presenting at one view, and according to real dimensions, every part of its surface, and the proportion and position of whatever it contains. That method is especially of use to many classes of artizans, as it enables them readily to compute the superficial amount of materials required for the completion of their work. In this method, every edge but one or two of a surface becomes disconnected from its contiguous faces, and laid flat or spread on a horizontal plane. It is evident, therefore, that as a tetrahedron (Fig. 75) is a rectangular figure, and composed of four equilateral triangles, one of which forms its base, three equilateral triangles, constructed one on each of the edges of that base, will show the development of the whole surface.

A cube or hexahedron, being composed of 6 square faces, will have the development shown in Fig. 76.

An octahedron consisting of eight equiangular and equilateral triangles, will be developed as in Fig. 77.

A dodecahedron, composed of twelve equilateral and

equiangular pentagons, will show its development as in Fig. 78.

An icosahedron, consisting of twenty equilateral and equiangular triangles, will be developed as in Fig. 79.

A cylinder, however, must be considered as a prism consisting of an infinite number of parallelogramic faces; therefore divide the circumference into any minute number of equal parts, and transfer each of those parts on a straight line by stepping the compass on it according to the number required. On account of want of space, the elevation of Fig. 80 only shows half the development of the cylinder.

A cone (Fig. 81), will be developed in a somewhat similar manner. From centre A, with radius AB, describe the indefinite arc BC, on the circumference of which step similarly the openings of as many sectors as the circumference forming its base is divided into.

These examples being understood, no difficulty will be experienced in the projecting Fig. 82.

A pentagonal pyramid 1 inch edge, 1.25 inch high, obliquely truncated at .7 of its height, having a portion of three of its contiguous faces cut by a vertical plane: and obliquely truncated towards its apex.

Show the development of a cone 1.9 inch diameter, and 1.125 inch high, showing the position of a circular, an elliptical, a parabolic, and a hyperbolic section. (Fig. 83.)

Divide the circumference into any number of equal sectors, *a b c d e f g h k*, and from these draw concentric lines to L. Draw the projectors aa'' bb'' cc'', &c., perpendicular to x y, and determine L′ the height of the axis of the cone. From L′ L as a centre draw the lines L′ a'', L b'' L c'', &c., to complete the elevation. From L′ as a centre, with radius L′ K, describe the arc a' k'', and take in the compass the distance a b and step it on the arc

a′ k′ a′ as many times as there are sectors in the circumference of the plan (16 times).

On the elevation, A′B′ will show the horizontal or circular section of the cone, C′D′ the position of the ellipse, *c″* E′ that of the parabola, and *f″* F′ that of the hyperbola. From these, their horizontal projection in AB, CD, *cc* E, and *ff* F, need scarcely be explained.

To determine their position on the development it is only necessary to draw horizontals from the generatrix through every point on the elevation, determined by the intersection of the sectional representation of the figure, with the vertical projection of the lines radiating from the vertex L′ to the base *a″ k″*; thus, for the development of the ellipse, the distances L′ 1, L 2, &c., transferred on the corresponding radii of the development, will determine the points through which the curve is to pass.

Similarly for the parabola, the horizontals from the sectional projection to the generatrix will also determine on the latter the corresponding points on the development. The hyperbola will follow the same rule.

SHADOWS.

A luminary is a body emitting a stream of light, rendering visible and illuminating the objects upon which it falls. Light proceeds in direct lines or rays, radiating when produced by an artificial luminary, such as a lamp or a fire or any other chemical or electrical agency, and forming a cone or pyramid of rays, increasing the size of the cast shadow of an interposing object according to distance, but decreasing in intensity and sharpness of form in the inverse ratio of its expansion. Except in particular cases, however, more within the province of the artist, this method of representing shadows is not practised. In Geometrical drawing, the great natural source of light, the

sun, is assumed to be the ordinary luminary; and for practical purposes, on account of its great distance from our sphere, its rays are supposed to fall parallel to each other, casting thereby the shadow of intercepting objects in a parallel direction on the planes influenced by them.

As that portion of an opaque surface directly exposed to the rays of light is illuminated, that portion which is opposed to it, and consequently deprived of its influence, is said to be "in the shade." The surface or space on which that shade is projected is called the "shadow." The terms "shadow proper," and "cast shadow," are sometimes applied to distinguish these two conditions.

According to nature, and with reference to the co-ordinate planes of projection, the projections of shadows are liable to vary indefinitely; for they may be influenced in plan by the assumed altitude and direction of the luminary, and in elevation by the given angle which the ray of light may happen to form with the vertical plane.

For the sake of uniformity, however, where no particular data is required, a conventional and universal method of determining shadows has been adopted by architects, and will be described hereafter.

The inspection of Diagram, No. 84, will explain the natural method of representing shades and shadows under various conditions.

Let the point *a* represent the position in plan, and the line *a′ a″* represent the elevation of a stick. Let *l* determine the vertical position of a luminary, and *l* its horizontal projection. In this instance it is evident that should the rays of light be parallel to the vertical plane, the shadow of *a′ a″* would be cast to *b′*, which, in plan, would be determined by a line proceeding from the direction of *l* through *a*, and determine the cast shadow *a b* equal in length to *a″ b′* in the elevation. If, from *a* as a centre with

radius *a b*, we describe a circle, its circumference will show the limits of the length of that shadow, no matter what angle the ray of light may form with the vertical plane on which the elevation is projected. Should the luminary be placed lower, as in *l′* 2, or higher, as in *l′* 3, the length of the projected shadow would evidently vary with the altitude of the luminary, for the latter placed in *l′* 2 would produce the shadow *a′*-*b* 2′, and, proceeding from *l′* 3, would determine the shadow *a′ b* 3′. Similarly the direction of the shadow in plan must vary with the position of the luminary in reference to the vertical plane. The luminary in *l* will give that shadow parallel to the vertical plane; *l* 2 will cast its shadow *a*-*b* 2, proceeding from the left front of the picture towards that plane, and *l* 3, proceeding from the left rear, will cast its shadow in *a*-*b* 3 towards the spectator. The distances *a*-*b*, *a*-*b* 2, *a*-*b* 3 forming, as we have already observed, the radii of circles determining the horizontal projection of the shadows. It is clear, therefore, that the given angle which the ray of light is to form with the horizon must be determined on the vertical plane, and that the angle which it makes with the vertical plane must conversely be determined on the horizontal plane.

These observations being understood, the following examples will not present much difficulty :—

1. Determine the shadow cast by a cube upon a horizontal plane, the rays of light forming an angle of 45° with the horizon, and parallel to the vertical plane. (Fig. 85.)

After having obtained the horizontal and vertical projections of the cube in the usual manner, determine on the vertical plane the angle *a″ b′ a′*, equal to the given inclination of the ray of light with the horizon: parallel to *a″ b′* draw *c′ d′* and *e′ f′*. On the horizontal plane make *a b* equal to *a′b′*, and parallel to it draw *c d* and *e f* of equal length. Join *d b f* to complete the shadow required.

N.B. The angle of 45° has the advantage of determining by mere inspection the height of an object, the plan only of which is given; the shadow at that angle being equal in length to the height of the object. The adoption of this property of the angle of 45° may be rendered especially valuable in military topography.

The shadow of an object being determined in plan, and the angle of inclination of the ray of light being known, it is likewise easy to determine its height, which equals that of the perpendicular ***a c*** of a right angle triangle ***a b c***, of which the length of shadow ***a b*** forms the base, and the hypothenuse ***b c*** forming with ***a b*** the known angle of inclination, forms the third side.

2. The shadow ***g b d*** of the pentagonal pyramid ***d e f g h***, and the angle of inclination which the ray of light makes with the horizon being given (60°), determine the height of the pyramid (Fig. 86). Join ***a b***, and draw indefinitely the perpendicular ***a c***: at ***b***, make the angle ***a b c*** equal to the given angle, and produce ***b c*** till it cuts ***a c***: ***a c*** will be the height of the pyramid.

3. Determine the shadow of a cylinder lying on its side, the rays of light forming an angle of 60° with the horizon, and of 40° with the vertical plane. (Fig. 87.)

The projections of the cylinder in this position are parallelograms similar to those that would be produced by a parallelopiped whose shadow ***a′ b′ c′ g′*** transferred on the horizontal plane, according to the given conditions, would produce the figure ***c d e f h***. But, as this assumed parallelopiped contains the cylinder whose diameters are in plan ***c c′*** and in elevation ***a′ a″***, their projection will be ellipses, the position of whose axis ***a b, g g″*** and ***k***, are determined by the direction of the angle of light. Inspection of the diagrams will show how to complete the figure.

4. Required the shadow of a cone projected on a wall behind it, the ray of light forming an angle of 30° with the horizon, and falling from the left front of the picture towards the wall at an angle of 55°. (Fig. 88.)

Determine the angle $a'\ b\ a''$ for the inclination of the ray of light to the horizon, and draw $a\ b'$ equal to $a''\ b$, forming with $x\ y$ an angle of 55°. Tangent to the base of the cone, and in the direction of b', draw $d\ e$ and $d'\ f$ for the horizontal portion of the shadow. Draw $c\ b''$ at right angles with $x\ y$, and make $b'\ b''$ parallel to it. Join $e\ b''\ f$ for the vertical portion of the shadow cast on the wall.

5. Determine the shadow of a square pyramid whose base is raised at a given height above the horizontal plane, the ray of light falling at an angle of 40° with the horizon, parallel to the vertical plane.

The mere inspection of Diagram 89 should suffice for the solution of this question, as well as that of Fig. 90, which represents—

6. A hexagonal prism of 1 inch side, and 1.2 inches edge, laid on one of its faces at right angles to the vertical plane.

PRACTICE.

7. Determine the shadow of a sphere cast on a horizontal plane, the ray of light forming an angle of 40° with the horizon, parallel to the vertical plane.

8. The axis of a cone 1 inch high and .75 inch diameter at its base, is at .65 inch distance from one side of a cubical box and at .6 inch from the other side: show the shadow which would fall both on the bottom and on the sides of the box, supposing parallel rays of light to fall upon the cone at angles of 30° with the horizon, and at 50° with the further side of the box. (Fig. 91.)

The inspection of this diagram will show its analogy

with Fig 88, from which it differs only by the addition of the auxiliary plane x' y', on which a portion of the shadow is projected according to the same method.

9. A line .5 inch long represents the plan of a stick 8 feet long, inclined in the direction of a wall 3 feet distance at an angle of 30°: show its shadow, the ray of light forming an angle of 45° with the horizon and with the vertical plane. (Fig. 92.)

Let a b represent the horizontal projection of the stick, having its upper extremity in a. To obtain its height, erect the perpendicular a c, and with radius equal to the whole length of the stick, from b as a centre determine the point C: transfer the distance a c to a' c' for the height of the vertical projection determined by the line c' b', which line may now be treated as the generatrix of a cone, and its shadow determined accordingly.

Inspection of the diagram will show that the question is but another partial application of question 4. (Fig. 88.)

In the conventional method adopted by architects, &c., it is assumed that the rays of light fall parallel to each other, in the direction of the diagonal of a cube, two of whose faces are parallel to the vertical plane, and the traces of which rays form with both planes angles of 45°. This combination of rays forms a real angle of 35° 16′ with the horizontal plane, and the shadows in the following diagrams are cast according to those conditions.

Let a b c d (Fig. 93) represent the plan, e f the elevation, and a c l m a section taken in the direction of the diagonal of the cube: it is evident that the line m c will show the direction of that diagonal (which has an inclination of 35° 16′ with the horizon) and that the ray of light l l', parallel to it, will determine on the horizontal plane the length of the shadow c l' cast by the edge of the cube c l. The edges b and d, being of equal height with

l c, will similarly cast their shadow, *b h* and *d k*: *h l'* and *l' k* drawn parallel to *b c* and *c d* will complete it.

On the elevation, determine likewise the shadow of the cube by making *e g* equal to *e f*, making thereby the angle *e g f* equal to 45°.

The rays of light cast on an object standing on a horizontal plane will produce direct rays of shadow; but if that plane is cut by another plane forming any angle with it or with the horizon, the projection of the ray of shadow will follow the direction of the intervening plane. Thus, in Fig. 94, to determine the projection of the shadow cast by a cube on a vertical plane at any given distance behind it. Let *a b c d* represent its plan, *e f g h* its elevation, and determine the ray of light *g k*, forming an angle of 45° with the horizon. Draw *b l*, *c m*, *d n*, equal to the diagonal *a c*, and forming likewise angles of 45° with *x y*. Join *b l m n d* for the projection of the shadow cast on the horizontal plane.

To show that portion of shadow which is cast on the vertical plane, at *o*, where *c m* intersects *x y*, erect the perpendicular *o p*, equal to *k m*, and join *k p*. At *s* draw *s t* perpendicular to *x y* and equal to *o p*, join *p t s*, and the whole shadow will be represented under the required conditions by the figure *b l k p t s d*.

Under similar conditions, determine the shadow cast by a square block, say 2 inches side, 1.5 inches high, surmounted by a square pyramid 3 inches high, whose base coincides with the upper surface of the block.

After having obtained in the usual manner the plan and elevation of the solid, determine the direction of the vertical and horizontal rays of light, as already explained, and, at the junction of the horizontal rays with *x y*, erect the perpendiculars, whose heights are determined by the same process as in the preceding example; from which the

present question is only a slight variation. The inspection of Diagram 95 should prove a sufficient explanation for its complete solution.

The mode of projecting the shadows described in the two last diagrams being well understood, no difficulty whatever need be experienced in treating any kind of prisms in a similar manner. Fig. 96, which represents the shadow cast under similar conditions by a given cone, may serve as a typical example for all like questions, whether relating to cones or pyramids.

The plan *a*, elevation *b*, and directions of shadow *c* and *d*, being determined, draw *e f* parallel to *c d* and equal to it. Join *g f* and *h f* for the vertical portion of the cast shadow. Draw *a k* and *a l* at right angles to *k g* and *l h* to determine the shade of the cone.

To project the shadow of a cylinder cast upon a wall under the same conditions. (Fig. 97.)

The plan and elevation of the cylinder being drawn according to the usual method, determine the angles of shadow. From centre A draw A *l g* produced, forming with *x y* an angle of 45°. From its vertical projection a' draw likewise the line a' *t*, forming the same angle with *x y*. Draw *t* a'' perpendicular to *x y* : a'' will be the projection of the point A, from which describe a circle equal to its projector. At right angles with A a' draw the diameter *b c*. Draw *b s* s' and *c t* t' tangent to both circles, in order to complete the horizontal projection of the shadow, that portion of which, however, lying in the horizontal plane is only required. That portion of the shadow which is projected on the vertical plane is determined thus:—

Parallel to *k* A *l* draw *d n*, and parallel to *d* d' draw *k b* b', *s a*, *n* d'', *e l* c', *e* e' and *g* g'. Parallel to a' *t*, from the points d' b' a' c', draw lines intersecting these in *o* d''' and g''', also in d'' and in g'. The point *u* is determined by drawing t' *u* parallel to *x y*. The visible part

of the curve is drawn through $o\ d''\ g'$ and u. The ellipse is completed by continuing it through d''' and g'''.

$b\ s\ o\ d''\ g'\ u\ t$ and c will determine the shape of the shadow required.

Determine the boundary of light on a sphere in horizontal and vertical projection; also the shadow projected by that sphere on both planes of projection, according to the same conditions.

Let A represent the plan, and B the elevation, of the sphere. Through the centre a and its vertical projection a' draw the line $b\ d$, $b'\ d'$, forming angles of 45°, and intersecting in d'. At right angles with these lines draw the diameters $e\ f$, $e'\ f'$, and draw the tangents $e\ p$, $f\ q$, $e'\ p'$, $f'\ q'$, parallel to the ray of light: join $e\ c$ and transfer the distance $e\ c$ from a to g, g will become the centre of the elliptical shadow of the sphere: through g draw $o\ g\ n$ parallel to $e\ a\ f$, for its minor axis.

Parallel to $e\ g$ draw $s\ r$ tangent to the plan of the sphere, and from the point s, through the centre a, draw $s\ k$, make $k\ t$ parallel to $e\ c$, and produce $q\ f$ in l, intersecting $k\ t$ in u. At u, the intersection of these two lines, draw $u\ m$ parallel to $e\ f$; the distanced $g\ m$ will equal half the major axis or the ellipse. Make $g\ v$ equal to $g\ m$, and through the points $m\ o\ v\ n$ describe the ellipse. At s draw $s\ i$ parallel to $e\ f$, the semi-ellipse $e\ i\ f$ will determine the limits of light cast on the sphere. Make $a\ z$ equal to $a\ i$, and the point z will determine the culminating point of light.

To represent the elevation, perpendicular to $x\ y$ draw $z\ z'$, $m\ m'$ $i\ i'$, $g\ g'$, intersecting $b'\ d'$. These points will correspond with those similarly lettered in the plan: thus $f'\ i'\ e'$ will determine the boundary of light in the elevation, z' its culminating point; the distance $g'\ m'$, produced in $g\ v'$, will complete the major axis, and $n'\ o'$, drawn through g, perpendicular to it, will determine the minor

axis of the ellipse formed by the shadow whose projections will be completed as shown in Diagram 98.

The following examples are mostly founded on constructions obtained from the "Traité de Dessin Industriel, par Messrs. Armangaud Jeune et Amouroux;" and a careful inspection of the diagrams will show the analogy between each and the method of construction adopted in similar cases.

Fig. 99 represents a cylinder 1.25 inches in diameter, surmounted by a cylindrical cap 2 inches in diameter and .25 inch high. In this, as well as in the following diagrams, the direction of the rays of light form the usual angle of 45° with both planes of projection; and, for the sake of convenience, only one half of the surface of each horizontal cap surmounting the solids is shown.

From centre A draw the radii A $c\,c'$ and A $e f$, at angles of 45° with the horizontal plane, and draw A $d\,d'$ at right angles to $p\,q$. Draw $b\,b'$, $d\,d'$, $e\,e'$, parallel to A $c\,c'$, and from each point thus obtained on both circumferences project the perpendiculars b'-b 3, c'-c 3, d'-d 3, e'-e 3, and from the centre b 2, with radius b 2-b 3, determine the distance b 2 b 4; from centre c 2, with radius c 2-c 3, determine the point c 4; from d 2, with radius d 2-d 3, determine the point d 4; and from the centre $c\,e$, with radius e 2-e 3, find the point e 4. A curve drawn by the hand through the points b 4, c 4, d 4, e 4 will show the shadow required. A careful inspection of the following diagrams will demonstrate that the mode of obtaining their shadows is precisely similar to that just explained, and therefore requires no further observations.

Fig. 100 represents an octagonal prism, surmounted by an octagonal cap of larger dimensions, and whose sides are parallel to the faces of the prism.

Fig. 101. A cylinder surmounted by a hexagonal cap.

Fig. 102. An octagonal prism, surmounted by a cylindrical cap.

Fig. 103. An octagonal prism, surmounted by a square cap.

Fig. 104. A square prism, surmounted by a cylindrical cap.

In these few examples on shadows I have purposely avoided entering into details which belong more to the province of the artist—the influence of reflected lights, of position, of distance, of the atmosphere, &c. The art of keeping or graduating lights or shades, under different circumstances, will be found in "Hints on Freehand Drawing," now preparing for publication.

The foregoing questions being understood, the solution of the following questions will be found easy.

Examples and Exercises on Shadows.

A pentagonal pyramid, 2 inches side and 3 inches high, rests on the plane of its base. Show its shadow when cast on the horizontal plane, the ray of light forming an angle of 40° parallel to the vertical plane.

Find the shadow of a frustum of a square pyramid, 3 inches high, and whose sides at the base are 3 inches long, and at top 2 inches. One side forms an angle of 30° with the vertical plane, and the ray of light falls at an angle of 60° with the horizon, and 45° with the vertical plane.

A cylinder, 3 inches long and 2 inches in diameter, lies on its side, which forms an angle of 30° with the vertical plane.

Project its shadow, the ray of light forming an angle of 50° with the horizon, and of 60° with the vertical plane.

A tetrahedron of 3 inches edge has neither of its sides parallel to the vertical plane, determine its shadow, the

angles of light forming 45° with the horizon and the vertical plane.

A cube of 2 inches edge has one of its faces forming an angle of 15° with the vertical plane. Draw its elevation, and also a section and sectional elevation on a plane, cutting two of the adjacent faces of the solid; project also its shadow, the rays of light forming angles of 30° with the horizon, and 48° with the vertical plane.

A tetrahedron of 4 inches edge rests on one of its faces. Draw its plan and a sectional elevation on any plane not passing through the apex nor parallel to either edge of the base. Draw the shadow, when the ray of light forms an angle of 35° with the horizontal, and of 27° with the vertical plane.

A hexagonal prism of 1 inch side has one of its faces forming an angle of 15° with the vertical plane. Draw a plan and section with shadow, the ray of light falling at an angle of 45° with the horizon, parallel to the vertical plane.

Required the shadow of an inverted cone 1.5″ high; diameter of the base 1 inch, raised .5 inch above the horizontal plane, direction of light 45° with the horizon, parallel to the vertical plane.

A stick 3 inches long, placed vertically, casts its shadow on two steps, and on a wall behind it at .5 inch, each step is .3 inch high and 2 inches broad; light according to conventional conditions.

Two cubes, the lowest of 1 inch edge, and the upper one 1.3 inches edge, are superposed, so that neither of their vertical faces coincide. The upper one is surmounted by a square pyramid 1 inch high, whose base coincides with the upper face of the cube, required their conventional shadow.

The profile section of a right prism is an equilateral

triangle of 37 feet side, the prism is 62 feet long, and rests on one face. Draw its plan of the plane of that face, and an elevation showing one face and one end of the solid. Also a sectional elevation on a plane parallel to its elevation. Give the shadow according to the conventional conditions. Scale $\frac{1}{120}$.

Required the plan, elevation, section and sectional elevation of a square pyramid 2 inches edge and 3 inches high. One of whose edges of the base forms an angle of 32° with the vertical plane. The vertical section is to cut three faces of the solid, not parallel to either side of the base, nor to pass through the vortex. Give also the shadow, the ray of light forming an angle of 50° with the horizon, and 25° with the vertical plane.

Let the above pyramid be truncated at $\frac{4}{7}$ of its height from the vertex: show likewise the shadow under similar conditions, and also the plan, elevation, and sectional elevation.

A prism 3.5 inches long has for its ends regular hexagons of 1 inch side, and stands on one of its faces. Show its shadow under conventional conditions, and a section on a plane making an angle of 30° with the side of the plan.

DESCRIPTIVE GEOMETRY.

As we have already observed, there is no universal system of projections. Objects may be represented orthographically in certain positions, and under certain conditions, by means of plans, elevations and sections. Their form may be determined by means of their traces, or projections, or through the agency of indices and contours. All these different methods, more or less based upon the same principles, have the same object in view, and tend to

the same end. In some cases the application of one method may be more convenient than another, and in many circumstances, in order to attain at the solution required, it is expedient to make use of a combination of the several methods.

Orthographic projections, the method of plans and elevations, being not only the most simple in construction, but also that which seems the more natural, and which speaks most readily to the eye and to the imagination, is that which generally comes under first notice. Under the heading of Orthographic projections, and with the view of suiting the case of a particular class of students, I have introduced a few cases requiring for their solution the application of certain problems of Descriptive Geometry before having mentioned the first principles of this most important part of geometrical drawing. If, however, with the assistance of their teachers, these anticipated cases were understood, the student will be better prepared for the apprehension of the following pages.

Descriptive Geometry was first introduced by Monge, Professor of Mathematics at the Ecole Polytechnique in Paris. It was he who first conceived and demonstrated that objects of three dimensions might be represented by means of their traces or projections, cast according to certain rules and conditions upon one single plane, representing in its turn two or more co-ordinate planes of projection. This important subject has since been enlarged and improved upon by several eminent mathematicians, until it has reached its present state of development.

Although some of the explanations and definitions given under the head of Orthographic projections (*vide page* 71) are in a great measure applicable to Descriptive Geometry, in as much that the position of the co-ordinate planes, the

mode of representing objects by means of their projections or traces, are mostly identical. It may not be useless, even at the risk of being prolix, to repeat those introductory definitions which are more applicable to this branch of drawing, and the perfect conception of which are absolutely necessary. I have neglected in this short treatise to repeat those introductory definitions relating to the plane and straight line, because they can be obtained in any copy of Euclid, Book XI. And it is hoped moreover that for pupils already partly trained in drawing, the inspection of the accompanying diagrams will be sufficiently explicit, and induce them to use their reasoning powers. If not, a few moments of ocular demonstration, of viva voce explanation from a competent instructor will have more efficacy than several written pages.

N.B.—In those kinds of demonstrations I have found an old stiff copy book cover, a few knitting needles, and some small corks most useful and convenient.

The following few elementary problems, relating to straight lines and planes, based mostly upon the original work, or on some of the free translations of "Lefebure de Fourcy," are fundamental, and may be considered as ranking amongst the most useful: a clear comprehension of them is recommended, as it will materially aid to the attainment of the present object, viz. to open the threshold, to guide in the path of a most useful branch of the science. The entire acquisition of which must, however, depend on the future efforts of the learner.

Definitions.

1. The projection of a point in space A, upon a given plane of projection B, is obtained by drawing a line A*a* from that point perpendicular to the plane. The point

where it meets that plane in *a* is the projecton of the original point A. (Fig. 1, Pl. XL.)

2. The projection of any line A B C D, &c. upon a given plane, will similarly be a line *a b c d*, &c., determined by the projection of every point in the given line upon the given plane. (Fig. 2.)

3. The projecting cylinder *a b* of a plane curve AB, perpendicular to the plane of projection, is a straight line, as its plane coincides with the plane of the curve. (Fig. 3.)

4. A straight line AB, forming any angle with the planes of projection, must be projected in a straight line *a b* (Fig. 4), since the projection of its extremities, *a b* are determined.

5. The position of a point in space A is determined when its projections *a* and *a′*, on two planes which intersect, are given. (Fig. 5.)

6. When two points on the co-ordinate planes, *a a′* or *b b′*, represent the projections of the same point in space A or B, the perpendiculars from these points, drawn to the line of intersection *x y*, must meet at the same points upon that line, *ab″ b″* (Fig. 6), and similarly a line connecting *a b* and *a′ b′* determines the position in space of the line AB.

7. A line is determined when its projections upon two co-ordinate planes are given; and, similarly, the projection of any curve, A B C, upon two intersecting planes of projection in *a b c* and *a′ b′ c′*, will determine that curve. (Fig. 7.)

8. The intersections of a plane with the two co-ordinate planes are termed the "traces" of that plane, *i. e.* the line in which a surface cuts the plane of projection is termed the trace of that surface. (Fig. 8.) Therefore *a b* is the vertical, and *b b′* the horizontal trace of the planes A *a′*, *b b′*.

9. A plane is determined when its traces are given. (Fig. 9.) Therefore *a b′* determines the vertical, and *b b′* the horizontal traces of a plane oblique to both planes of projection.

10. We have already mentioned (*vide* Fig. 1, Pl. 17) that the angle contained by two planes of projection is ever supposed to be a right angle. The upper plane is termed the "vertical plane," and the lower plane the "horizontal plane." Their line of intersection is termed "line of level," or "ground line;" but, in order that one single plane may contain both planes of projection, the vertical plane is supposed to turn round the intersection of the ground line until it coincides with the level of the horizontal plane. The term "draught" is given to the drawing containing the solution of a problem. The data and quaesita are represented by lines of various thickness, and the constructions by dotted or broken lines of different kinds.

The original points (very seldom seen in the draught) are determined by the capitals A B C, &c., their horizontal projections by italics, *a b c*, &c., and their vertical projections by the same kind of letters accentuated, as *a′ b′ c′* &c. When these points are situated in the gronnd line *x y* the Greek letters α β γ, &c., usually determine their position.

When a point, a line, or a plane, are said to be known, it signifies that their projections are known, and, conversely, when they are to be determined, it is sufficient to determine their projections.

Problems.

1. Given the projections *a a′ b b′* of a line above *x y* and in front of the vertical plane, to determine its traces and real position. (Fig. 10.)

If we conceive a vertical plane passing through *a b*, the distance *b b′* being equal to the height of *b* above the vertical plane, *b′* will be its vertical trace, and if, similarly, we conceive a second plane, perpendicular to the vertical plane, passing through *a′ b′*, the distance *a′ a*, being equal to that of *a* from the vertical plane, will give its horizontal trace in *a*.

It is clear, therefore, that a line joining *a b′* would give the given line.

2. When the horizontal trace *a b* is in front of *x y*, and the vertical trace *a′ b′* in the lowest part of the vertical plane. (Fig. 11.)

c c′ drawn through *x y* in the prolongation of the vertical plane, and *b b′* drawn perpendicularly to *x y*, meeting *a′ c′* produced in *b′*, will determine the real position of the line.

3. When the horizontal trace is behind *x y*, and the vertical trace is above it. (Fig. 12.)

Produce *a c* indefinitely, and draw *c c′* perpendicular to *x y*, at *b* draw the perpendicular to *x y* cutting *a c* produced in *b′*.

4. When the horizontal trace is behind *x y*, and the vertical trace is in the lower part of the vertical plane. (Fig. 13.)

Produce *a c* indefinitely in *b′*, and produce likewise *b c′* in *c′* : draw *c c′* and *b b′* perpendicular to *x y*.

5. Given the traces of a line *a a′ b b′*, to find its projections.

Join *a b* and *a′ b′* for the projections required. (This is the converse of Prob. 1, Fig. 10.)

6. *Vide* Prob. 18, Fig. 25.

7. Through a given point *a a′* to draw a line parallel to a given straight line, whose projections are *b c b′ c′*.

Join *a a′*, and through these points draw *d e d′ e′* parallel to *b c b′ c′*. (Fig. 14.)

The simplicity of this problem renders further explanation unnecessary.

8. To find the intersection of two planes whose traces are *a b b′* and *c b b′*. (Fig. 15.)

Draw *b′ d′* and *b d* perpendicular to *x y*. Join *b′ d* and *d″ b*, which are the projections of the intersecting line *b b′* required.

Now imagine two oblique planes intersecting each other, and the two planes of projection in *a b b′* and *c b b′*, a line joining *b b′* would evidently determine the meeting of the oblique planes: hence the vertical projection of the intersection is in *d′*, and the horizontal projection in *d*.

9. Given the traces of a plane *a b a′ b′*, to find their projections when they are parallel to the ground line. (Fig. 16.)

Draw any line *c d c′* perpendicular to *x y*, as the trace of an auxiliary plane which is to contain the profile angle. From *d* as a centre, with radius *d c*, describe the arc *c e*, then the angle *d e c′* will show the inclination, and *e c′* the length of the given plane.

10. To find the projection of the common section of two planes *a c a c′*, when one of the planes has its trace perpendicular to the ground line and the other side oblique to it. (Fig. 17.)

The trace *a a′* being perpendicular to *x y*, and *a c* being oblique to it, the line of intersection will evidently be in *a a″ a′*.

11. To find the projection of the intersection of two planes inclined towards each other when their traces are parallel. (Fig. 18.)

Let the two parallel planes *a p a′* and *b q b′* intersect in S′. Draw S S′ perpendicular to *x y*, and *s′ r′* parallel to it. Draw likewise S *r* parallel to *a p* and *b q*, then *s r*, *s r′* will be the traces of the intersection of the two planes.

It is clear that the intersection of two planes which are parallel at their base and inclined towards each other must be in a line parallel to those bases.

12. To find the projection of the common section of two planes $a\ a'\ a''$, $b\ b'\ b''$ when their traces meet the ground line in the same point. (Fig. 19.)

Determine the auxiliary plane $g\ o\ g'$, perpendicular to $x\ y$. From O as a centre describe the arcs $a'\ a''$, $b'\ b''$, and join $a\ a''$ and $b\ b''$: through their point of intersection c draw $c\ c'$ parallel to $g\ o\ g'$, and $c''\ d$ parallel to $a''\ o$, and from O as a centre describe the arc $c\ d'$. The lines $d\ h$ and $d'\ h$ determine the projection of the two planes.

The projections of the planes being oblique to each other, and to the planes of projection, their line of intersection must likewise be oblique.

13. To find the projection of the common intersection of two planes, when their traces are parallel to $x\ y$. (Fig. 20.)

The solution of this problem is very similar to that of the two former. Careful inspection will show their analogy to each other.

The traces of the planes being parallel to $x\ y$, the traces of their common intersection must likewise be in a line parallel to them.

14. To find the intersection of a given line with a given plane.

An assumed auxiliary plane, passing through the line, and cutting the given plane through its intersection with the given line, will give the solution required. As a first case, let us assume the plane passing through the given line to be perpendicular to the horizontal plane, and let $a\ a\ a'$ be the given plane, and $b\ c$, $b'\ c'$ the given line. (Fig. 21.)

Produce $b\ c$ to m; draw $m\ m'$ and $d\ d'$ perpendicular to $x\ y$; join $d'\ m'$ (for the projection of the auxiliary plane

intersecting $b\ c'$ in o); join $o\ o'$ perpendicular to $x\ y$ for the points of intersection.

Should the auxiliary plane be perpendicular to the vertical plane, join $n\ n'$ and $e\ e'$ perpendicular to $x\ y$, and draw $e\ n$ intersecting $b\ c$ in o.

15. The assumed auxiliary plane passing through the given line may have any position whatever. (Fig. 22.)

As the traces of that plane must pass through those of the line, determine the traces of the line by producing them to m and n. From any point β draw $\beta\ p\ \beta\ p'$ for the traces of the plane intersecting $a\ a'$ in e', and draw $e'\ e$ perpendicular to $x\ y$. Join $d\ e$ and $d'\ e'$; their intersection with $b\ c$ and $b'\ c'$ will give $o\ o'$ the points of intersection required.

16. When the given line is perpendicular to either of the planes of projection—say to the horizontal plane. (Fig. 23.)

Let $o\ o''$ be the projections of the line. Draw $d\ d'$ perpendicular to $x\ y$ for the vertical traces of the auxiliary plane whose trace is in $d\ o\ c$. Join $c\ c'$ perpendicular to $x\ y$, and join $c'\ d'$ intersecting o'' in o'.

The solution of this problem may also be obtained by drawing $o\ e$ parallel to $a\ a$, or $o\ f$ parallel to $x\ y$, and $f'\ o'$ parallel to $a\ a'$, and $e'\ o'$ parallel to $x\ y$. Either case will determine $o\ o'$.

17. A plane whose traces $a\ a\ a'$, and a point whose projections $b\ b'$ are given: to find the projections of a straight line drawn from the given point perpendicular to the plane, and to determine likewise the projections of the point of intersection of this line with the plane. (Fig. 24.)

Draw $b\ c$ and $b'\ c'$ perpendicular to $a\ a\ a'$. Produce $b\ c$ to $x\ y$ in f, and draw $f\ f'$ perpendicular to $x\ y$. Draw $e\ e'$ perpendicular to $x\ y$, and join $f'\ e'$. $c\ c'$ drawn perpendicular to $x\ y$ will determine the point of intersection of the line with the plane.

The analogy of this problem with those preceding it renders further explanations unnecessary.

18. To determine the real length of a straight line $a\ b$, $a'\ b'$ whose projections are oblique to both planes. (Fig. 25.)

Through a' draw $c\ d$ parallel to $x\ y$, and make $c\ f$ equal to $a\ b$. Join $b'\ f$ for the real length of the line, or conversely draw $b\ h$ perpendicular to $a\ b$ and equal to $c\ b'$. Join $a\ h$ for the true length of the line.

19. The projections of three planes, $a\ b\ b'$, $c\ d\ d'$, $e\ f\ e'\ f'$ being given, to find their points of intersection. (Fig. 26.)

Determine first, by their traces (Prob. 10), the intersection of any two of the planes—say $a\ b$ in b'' and $a\ b'$ in b''' with $c\ d$ in d'' and d' in d'''. Determine likewise the intersection of the third plane with either of the former—say that of $e\ f$ in f'' and f' in f''' with $c\ d$ in d'' and d' in d'''.

The points $o\ o'$ passing through the intersection of the traces in a line perpendicular to $x\ y$, determine the point of common intersection of the planes.

20. Having a plane given by its traces $a\ b\ a'$, and a point whose projections are $d\ d'$, to find the traces of a second plane passing through the given point and parallel to the given plane. (Fig. 27.)

Draw $d\ e$, $d'\ e'$ parallel to $x\ y$, and draw $d\ k$ and $d'\ g$ parallel to $a\ b$ and $b\ a'$: join $e\ g$ and $e'\ k$: through e and e' draw $l\ m\ m\ l'$ parallel to $a\ b\ a'$; these two lines will be the traces of the second plane required.

Since the position of the point is given by its traces $d\ d'$ and the required plane passing through this point must be parallel to $a\ b\ a'$, $d\ k$ drawn parallel to $a\ b$ will determine its horizontal projection, and $k\ e'$ equal to $d'\ d''$ will give its altitude. In the same manner $d'\ g$ drawn parallel to $b\ a'$, and $g\ e$ perpendicular to $x\ y$ and equal to $d\ d''$ will determine its horizontal trace: $m'\ l'$ drawn through e' will therefore be the vertical trace of the second parallel plane, and $m\ l$ drawn through e will give its horizontal trace.

21. The traces of a plane *a b a′*, and also one of the projections of a line *c d* in that plane being given, to find the other projection. (Fig. 28.)

Draw *d d′* perpendicular to *x y* for the vertical trace of the plane passing through *c d*, and draw *c c′* perpendicular to *x y*. The line *c′ d′* will be the vertical projection of the line required.

22. Given a straight line whose projections are *a b* and *a′ b′* and a point whose projections are *c c′*, to construct the traces of a plane passing through the point and perpendicular to the straight line. (Fig. 29.)

Draw *c d* perpendicular to *a b*, and *c′ d′* parallel to *x y*; draw *d d′* perpendicular to *x y*, and through *d′* draw *f′ g* perpendicular to *a′ b′* for the vertical trace of the required plane: *g f* drawn parallel to *c d*, and consequently perpendicular to *a b*, will determine its horizontal trace.

This problem, being a combination of problems 17 and 20, requires no further explanation.

23. To draw a plane through three given points *aa′*, *bb′*, *cc′*. (Fig. 30.)

Join *a b b c* and *a′b′ b′c′*; produce *a b* to *d*, and *c b* to *f* in *x y*. Produce also *b′ a′* to *g′* and *b′ c′* to *k′* in *x y*: join *a a′*, *b b′*, *c c′* and draw *d d′* and *f f′* perpendicular to *x y*: draw *g g′* and *k k′* perpendicular to *x y*, the lines *d′ f′* and *k g* will be the required traces of the plane. Three points in any position can always be contained by one plane, therefore the three given points will in this instance determine that plane: the vertical projections *b′ c′* and *b′ a′* produced to *k′* and *g′* in *x y* will give the vertical projections of the base of the plane containing them: the horizontal projections *a b*, *b c* produced to *d* and *f* in *x y*, will give the horizontal projections of their altitude; *f f′* and *d d′* drawn perpendicular to *x y* and cutting *c′ b′* and *a′ b′* produced in *d′ f′* will determine therefore the vertical

projection of the plane, and similarly *k k′* and *g g′* drawn perpendicular to *x y*, cutting *a b* and *b c* produced in *k* and *g*, will determine the horizontal projection of the plane containing the three points.

24. To draw a plane through a given point *a a′* and through a given line *b c*, *b′ c′*. (Fig. 31.)

Draw *a d* and *a′ d′* parallel to *c b* and *c′ b′* for the projections of the given line, *c* and *e* will be the horizontal and *b′ d′* the vertical traces of the line: the lines *c e* and *b′ d′* will be the traces of the required plane, and if produced in *x y* will meet at the same point *g*.

25. To draw a plane through a given line, parallel to a given line. (Fig. 32.)

Let *a b*, *a′ b′* be the projections of the first line, and *c d* *c′ d′* the projections of the second line: in *a b*, *a′ b′*, take any point *f f′* and through *f* draw *g h* parallel to *c d*. Draw *h l* perpendicular to *x y*, meeting *k f* in *l*: draw *k g* perpendicular to *x y*, intersecting *h f* produced in *g*, lines drawn through *g a* and *b l* produced till they meet in *x y* will give the traces of the required plane.

26. Through a point *e e′* given by its projections, to draw a plane parallel to the two given lines *a b*, *c d* and *a′ b′*, *c′ d′*. (Fig. 33.)

Through *e* draw *f h* parallel to *a b*, and *k l* parallel to *c d*; through *e′* draw *n o* parallel to *a′ b′*, and *l m* parallel to *c′ b′*, join *h m* and *o k* and through *k* and *m* draw *f p n*.

27. From a given point *a a′* to draw a line perpendicular to a given plane *b c b′*, and to determine its magnitude. (Fig. 34.)

Through *a a′* draw *a d a′ d′* perpendicular to *b c b′* and determine their projections through *d′*, draw *d e* parallel to *x y* and equal to *a d*: join *a′ e* for the length of the line required. (*Vide* Prob. 18.)

28. Through a given point *a a′* to draw a line which

shall meet two given lines $b\,c$, $b'\,c'$ and $d\,e\,d'\,e'$. (Fig. 35.)

Draw $g\,f$, $g'\,f'$ for the projections of a plane passing through the given point. Let $f\,a\,g$ represent the traces of the first line, and $f\,\beta\,g$ the traces of the second line; $g\,f$ and $g'\,f'$ will be the projections of their intersection.

In order that the construction should be correct it is necessary that the intersection should pass through the given point, and meet both given lines.

29. Through a given point $a\,a'$ to draw a line and a plane which will both be perpendicular to a given line $b\,c$, $b'\,c'$. (Fig. 36.)

Draw $a\,d$ perpendicular to $b\,c$ and $a'\,d'$ parallel to $x\,y$; draw $k\,m'$ perpendicular to $b'\,c'$ and $k\,m$ perpendicular to $b\,c$, then $m\,k\,m'$ will be the traces of the plane required.

Find $p\,p'$ for the projections of the intersection of the line with the plane, and draw $a\,p$, $a'\,p'$ for the projections required.

30. The projections $a\,b$, $a'\,b'$ of a line being given, to find the angles which it makes with the planes of projection. (Fig. 37.)

Determine the traces $a\,a'$, $b\,b'$ of the given line, and on $x\,y$ make $m\,b$ equal to $a\,b$, and join $m\,b'$ for the real length of $b'\,a$—the required angle will evidently be $b'\,m\,b$.

If it is required to determine the angle on the horizontal plane draw $b\,k$ perpendicular to $a\,b$ and equal to $b\,b'$, $k\,a\,b$ will be the required angle.

If we wish to find on the horizontal plane the angle which the line $a\,b'$ makes with the vertical plane, from centre a', with radius $a'\,b'$, describe the arc $b'\,s$ and join $a\,s$: then $a\,s\,a'$ will equal that angle. If we wish to determine it on the vertical plane, from a' as a centre, with radius $a'\,a$, draw the arc $a\,n$, $a'\,n$ being perpendicular to $a'\,b'$ then $a'\,b'\,n$ will be the required angle.

31. To construct the angle contained by two planes whose traces are $a\,b\,b'$ and $c\,b\,b'$. (Fig. 38.)

Draw $b'\,e$ perpendicular to $x\,y$ and join $b\,e$: draw anywhere $g\,k\,h$ perpendicular to $b\,e$: from centre e, with radius $e\,k$, describe the arc $k\,k'$, and with the same centre, with radius $e\,b$, describe the arc $b\,l$. Join $b'\,l$ and draw $k'\,m'$ perpendicular to it. Make $k\,m$ equal to $k'\,m'$, and join $g\,m\,h$, which will show the dihedral angle contained by the two planes and reduced to the horizon.

In this problem it is evident that the profile angle of the two planes will form a right angled triangle, of which the perpendicular will be $e\,b'$ and the base will be $e\,b$: that base $e\,b$ being transferred in $e\,l$ on $x\,y$ will determine the angle $e\,l\,b'$ on the vertical plane. As $g\,k\,h$ determines the base of the section the distance $e\,k$ transferred to $e\,k'$ will determine its position on $x\,y$, and $k'\,m'$ drawn perpendicular to $b'\,l$ will determine the height of the section: $k'\,m'$ being reduced to the horizon in $k\,m$, and $g\,m\,h$ being joined will show the dihedral angle contained by the two planes.

32. Two planes are inclined at angles of 50° and 67° respectively, and intersect in a line inclined at an angle of 35° to the horizon. Find the dihedral angle contained by them. (Fig. 39.)

Draw $e\,a$ perpendicular to $x\,y$, and from a make the angle $e\,a\,b$ equal to 23°, $e\,a\,c$ 40° and $e\,a\,d$ 55°; complements of the angles $a\,b\,e$ 67°, $a\,c\,e$ 50°, and $a\,d\,e$ 35°, according to the given conditions. From e as a centre, with radius $e\,b$, describe the arc $b\,b'$; with the same centre, with radius $e\,c$, describe the arc $e\,c'$; draw $d\,b'$ and $d\,c'$ tangent to those arcs for the horizontal traces of both planes.

To obtain the dihedral angle, at right angles to $x\,y$ draw $g\,h$ intersecting the two horizontal traces $d\,b'$ and

d c′; draw *k l* perpendicular to *d a*, and from *k* as a centre with radius *k l*, describe the arc *l m*: join *m g*, *m h*, and the triangle *g m h* shows the dihedral angle required, reduced to the horizon.

The student will see the analogy between the latter part of this problem and the preceding one.

33. The traces *p q r* of a plane being given, to find the angles which the plane makes with the two planes of projection, and also the angle between the traces. (Fig. 40.)

1st. Find on the vertical plane the angle the plane makes with the horizontal plane.

Draw *b c′* perpendicular to *x y* and *b a* perpendicular to *p q*; from centre *b*, with radius *b a*, describe the arc *a v*, join *v c′* and the angle *c′ v b* is the angle required.

2nd. To find the same angle upon the horizontal plane.

Perpendicular to *a b* draw *b c* equal to *b c′*, and join *a c*: *c a b* will be the angle required.

To find on the vertical plane the angle which the plane makes with the vertical plane, draw *b e* perpendicular to *x y* and *b d* perpendicular to *q r*, make *b t* equal to *b e*, and *t d b* will be the required angle. To find the same upon the horizontal plane: on *x y* make *b z* equal to *b d*, *z e b* will be the angle required.

To find the angle between the traces.

As the hypothenuse *a c*, to the right angle triangle *c b a*, determines the length of the plane *a c′ q*, if we suppose that plane to revolve on *a q* till it is reduced to the horizon and developed in *a a″ q*, the angle *a q a″* will give the angle of the traces.

34. To find the angle contained between a plane and a straight line. (Fig. 41.)

Let *a b b′* be the traces of the plane, and *c d*, *c′ d′* the projections of the line. If from any point on the line a perpendicular is drawn to the plane, the angle contained

by the given line and that perpendicular will be the complement of the required angle; therefore, from any point *a a'* on the line draw *c c'* perpendicular to *x y*, and draw *c e c' e'* perpendicular to *a b* and *a' b'*, construct the angle between the two lines, which will be *d e f*; draw *f k* perpendicular to *f e*, and the required angle will be *d f k*.

35. To construct the angle contained by two straight lines, intersecting in space, and whose given projections are *a b c* and *a' b' c'*. (Fig. 42.)

Produce *a' b'* and *a' c'* to *d'* and *e'* in *x y*, and draw *d d' e e'* perpendicular to *x y*. Produce likewise *a b a c* to *d* and *e*, and join *d e*. Draw *a a'* perpendicular through *x y* and *h g*, perpendicular to *d e*. Make *f k* equal to *a h'*, and join *a' k*: make *h g* equal to *k a'*, and join *d g*, *g e*. The angle *d g e* is the angle required, reduced to the horizon.

36. To reduce a given angle to the horizon, having given the angles which its sides make with it. (Fig. 43.)

Let *a* represent the horizontal projection of the vertex, *a b* the horizontal projection of one of the sides, the projection of the side *a e* is required.

Draw indefinitely the perpendicular *a a'*, and make the angle *a b d* equal to that of the first side (say 40°.)

Draw *d c*, so that the angle *a c d* may equal that made by the second side with the horizon, (say 55°.) From *a*, with radius *a c*, describe indefinitely the arc *c f*, and make *c d g* equal to the given vertical angle (say 42°.) Draw *b g* perpendicular to *x y*. From *d*, with radius *d c*, describe the arc *c g*.

From *b* as centre, with radius *b g*, determine that distance in *e*, intersecting the arc *c f*. Join *a e*: *a e* will determine the horizontal projection of the side required, and the angle *a e b* will be equal to the observed angle reduced to the horizon.

HORIZONTAL PROJECTIONS.

Topographic, horizontal, or, as it is sometimes termed, single projection, is a sister branch of Descriptive Geometry, which enables us to determine and ascertain by means of numbers or indices on one single ground plan, the position, shape, and altitude of the objects it contains. That method is very convenient for civil and military engineering purposes, and most especially applicable to military topography, whether for illustrating the accidents and irregularities of large tracts of country, or for determining the position, shape, and relief of military works, as well as the importance they bear to each other: for that method enables us, with the help of Descriptive Geometry, to construct elevations, sections, &c., of the planes containing these works, either for the purpose of defilading them, or for other operations.

Indices are numbers refering to the altitudes of points above the horizontal plane. That plane is usually assumed to be on a level with points (0 or zero) below the lowest parts of the plane. Therefore on a plan, a point whose index is 30, signifies that it is 30 units above the Zero plane, or lowest plane of level.

In some instances military men find it more convenient to assume a plane parallel to the horizon, and passing at some distance above the highest point of ground shown on the plan (usually 10 yards). In this case the indices show a decrease of height below that plane instead of an increase above the Zero plane. In the following problems we will, however, follow the first method.

As the position of points are determined by their projections, the letters, numerals, or indices which accompany them denote also their height. Their notation follow the rules already given.

In a similar manner, the position of a straight line will be determined by the projection and indices of its two extremities. (Fig. 1.)

Let, for instance, *a* 3 and *b* 6 represent the two extremities of the line *a b*, it is clear that the point *b* will be 3 units higher than the point *a*, that being the difference of height between *a* and *b* above the horizontal plane. Erect, therefore, at *b* the perpendicular *b c*, equal in height to the difference of units between the indices 3 and 6, and join *a c*, which will determine the ***slope*** or ***inclination of the line***, *b c* will be the ***scale of the line***, obtained according to the conventional scale of the drawing, and *a b* will be the ***scale of slope or inclination.*** The divisions on the ***scale*** of slope or inclination, and those on the ***slope***, or inclination, are always determined by those on the scale of the line. To complete the scale, through 4 and 5, draw the horizontals 4-4′ and 5-5′ parallel to *a b*, then draw the perpendiculars 4′ 4′′ and 5′ 5″.

Now, suppose the triangle *a b c* to revolve on *a b*, as on an axis, until *c* is perpendicular to the plane of the paper. The real position of the line *a b* will be determined.

N.B. When scales of slope or inclination refer to lines, they are represented by single straight lines. When they refer to planes they are shown by double lines. (Fig. 2.)

As one index determines the position of a point, two indices that of a line, we will find that three indices can determine the position of a plane.

A plane is represented by its trace on the horizontal plane of comparison, and by its inclination to that plane.

As the scale of the plane is that which determines its angle of inclination by its altitude between two given points, a plane is likewise determined by a vertical trace showing its angle of slope or declivity.

The comparative altitudes of the several parts of a plane are often shown by horizontal contours.

Contours are equidistant horizontal planes supposed to pass through the object, parallel to the plane of comparison, and showing the respective height of the different parts of the object above the horizon.

Let *a* 3, *b* 15 and *c* 12, not in the same straight line, determine the position and altitude of the several points of a plane above the plane of comparison. (Fig. 2.)

Join *a b*, the two extreme indices, and divide that distance into as many units as are contained in the difference between *a* and *b* (12) from *c* 12, draw a straight line, cutting *a b* at the point similarly indicated on it; *c* 12 and *c′* 12 becoming therefore points on the same level. These will necessarily indicate the position of one horizontal or contour of the plane containing them: to complete the plane, through 5, 6, 7, 8, 9, 10, 11, 13, 14 and 15, draw lines parallel to *c* 12, *c′* 12, and the direction of the levels of the plane will be defined. To find the scale of inclination of that plane draw any where the line *d e* perpendicular to *c* 12, *c′* 12, and make *e f* perpendicular to *d e*, and equal in altitude to the difference of units (by conventional scale) between the indices *a* and *b*. Join *d f*, and the angle *e d f* will be the profile angle or angle of greatest inclination of the plane *a b c* : *e f* will become the scale of the plane, and *d e* its scale of inclination. If we conceive the triangle *d e f* to revolve on *d e* till it stands perpendicularly to the plane of the plan, and another plane intersecting them perpendicularly through the diagonal *d e*, and the horizontals of the plan transferred to them, we will obtain a clear conception of the position of the inclined plane.

As a plane can be determined by three points, the indices of a triangle will evidently determine likewise its scale and inclination.

When a plane is horizontal its traces and indices represent its position. When vertical it is determined by a single index.

The scales of inclination of the plane being always constructed on the horizontal plane, it is evident that the position and length of two or more scales of inclination being given, we can easily obtain the line of intersection of these planes by producing their horizontals till they meet. Fig. 3 represents the intersection of the different planes of an irregular solid, all meeting at a common apex; and the different altitudes of which are determined by horizontal contours.

This diagram shows how the different parts of the undulating surface of a district can be approximately reduced to figures somewhat of a similar kind, how the indices and direction of contours may be used to determine the flexures, accidents, steepness of slopes and dispositions of the surfaces whose general characteristics are required.

These preliminaries being understood, the construction of the following problems will be found simple and easy, and their application to some of the questions which have hitherto been given, will become evident.

1. To determine the length and inclination of a given straight line 1.9 inches long, whose indices are *a* 5 and *b* 20. (Fig. 4.)

Draw *b c* perpendicular to *a b*, and equal to the sum of the difference between the indices (10), join *a c*, the length required.

2. To determine the horizontals and inclination of a plane containing three given indices, say 16, 23 and 21. (Fig. 5.)

This problem having already been explained, needs no further demonstration.

3. The first and last indices of two horizontals, which are parallel to each other, and 1 inch apart, are respectively 5 and 12: show their slope and scale of the plane. (Scale 10 units to 1 inch.)

The construction of this problem depends on the same principles as the former.

4. Two horizontals, *a b* and *a' b'*, are parallel and 1 inch apart. They represent respective heights of 57 and 73 yards above the ground. Find the position of a point 63 yards high. (Fig. 6.)

Draw any line at right angle with the horizontals, and divide it into as many parts as there are in the difference between the two indices (16). Number these, and through the resulting index (63) draw a horizontal parallel to *a b a' b'*.

N.B. In cases of this kind, the sector can be conveniently applied: to this effect, take in the compass the length *c d*.

(The point 63 being at $\frac{6}{16}$ or $\frac{3}{8}$ of the indices 57 and 73) adapt it to the distance 8-8 on the line of lines, and close the compass so as adapt it to 3—3, which length, transferred from *c* towards *d*, will determine the required height.

5. The length and extreme indices of the scale of inclination of a plane being given (say 7 and 21) to determine the height of any point or points.

Let *a b* be the length of the given scale, *c* and *d* the points whose altitude are to be determined. (Fig. 7.)

Divide *a b* into 14 equal parts (the number of units between) the two extremities of the scale 7 and 21. From *c* and *d* draw perpendiculars to *a b* for horizontals, whose heights will be determined by the number on the scale with which they coincide.

6. To determine whether a given line does or does not lie in a given plane. (Fig. 8.)

Two lines in plan may cross each other without necessarily meeting.

The lines ***a b*** and ***e f*** meet because their indices are the same at their point of intersection, and a line drawn perpendicular to their elevation will show the same result in both planes.

The lines ***c d*** and ***e f*** do not meet because neither their indices in plan or elevation coincide.

7. To determine the common intersection of two planes when the scales of inclination are parallel. (Fig. 9.)

Given the scales ***a b*** and ***c d***.

Make the height ***b e*** equal to the number of units indicated on the scale.

Through ***f*** on ***a b*** draw the perpendicular ***f h*** equal to ***b e***, join H***k*** and draw ***k c*** perpendicular to ***c d***; join ***e a***, intersecting ***h k*** in ***l***.

Through ***l*** draw ***l m*** perpendicular to both scales. The intersection of the two planes will be determined by the indices cut by ***l m*** at 7.

8. To determine the intersection of two planes when their scales are oblique to each other. (Fig. 10.)

Through points in the scales similarly numbered, say 12-23, draw perpendiculars to the scales: through their points of intersection, ***c*** and ***d***, draw the intersecting line ***c d***.

Should the numerals of the scale increase in value as they come nearer to each other (as in the figure) the line of intersection would show the ridge. Should they on the contrary decrease in value as they approach, the line of intersection would denote a furrow.

Should we require a profile section of the meeting of the two planes, the line ***d e***, perpendicular to ***c d***, and equal in altitude to the difference of the two indices (12 and 23)

would give the height, and the line $c\ e$ would show the inclination of the section required.

9. Construct a section showing the intersection of a vertical plane with an inclined plane. (Fig. 11.)

Given $a\ b$, the scale of slope of the plane, and $c\ d$ the horizontal trace of the vertical section required.

From any indices on the scale (say 7 and 14), draw perpendiculars $c\ e$ and $d\ g$, equal in height to the numbers or indices affixed to the scale, the line joining $e\ g$ will give the section required.

10. Two horizontals, $a\ b$, $c\ d$ 1.1, inch apart represent the horizontal projections of a plane inclined at an angle of 28° to the horizon, and containing a line 1.5 inch long. Show its plan. (Fig. 12.)

$a\ b$ and $c\ d$ are the two given horizontals. Make the angle $a\ c\ a'$ equal to the given angle, and produce $a\ c$ in h. From a' as centre, with a radius equal to the length of the line to be contained by the plane (1.5″) determine on $a\ h$ the point e. From a as a centre, with radius $a\ e$, describe the arc $e\ g$, and join $a\ g$, the horizontal projection of the line contained by the plane.

11. A plane, determined by two horizontals, 1 inch apart, is inclined at an angle of 50° to the horizon, and contains a line inclined at an angle of 30 degrees with the same, determine the length of the line and its horizontal projection. (Fig. 13.)

(The last problem should point out the solution of this.)

12. Determine the intersection of two planes whose traces form with each other angles of 80°, and whose slopes are respectively at angles of 40° and 60°.

Draw two lines, $a\ b$ and $b\ c$, meeting at the given angle.

Draw any where the perpendiculars $b\ f\ b\ g$.

Make *b d* and *b e* of any equal height, or of a given altitude.

Make the angle *b d f* equal to 50°, complement of its inclination of slope, and also the angle *b e g*, equal to 30°, the complement of the second angle of slope.

Through *f* and *g* draw *h k* and *k l*, parallel to *a b*, *b c*, the angles *b f d* and *b g e* will show the profile angles of the planes. Join *b k*, and draw *b m* perpendicular to it, and equal to *b d*, join *m k*: the angle *b k m* will show the profile angle at the meeting of the slopes. (Fig. 14.)

13. A hill slopes at an angle of 1 in 5: show on 6 contours the position of a path having an inclination of 1 in 10 with the horizon.

Assume the contours at .25 of an inch interval, each contour representing an increase of 10 feet in height. Scale 200 to 1 inch.

Six horizontals will represent 5 heights of 10 feet, or 50 feet. Construct a profile elevation on *a b*, making the perpendicular *a c* equal to 50 feet.

As each contour shows a rise of 10 feet, the plan of each space between the horizontals will contain a line 10 times the length of 100 feet long, or a total of 500 feet long, representing the horizontal projection of the path *b d*, whose section is determined by the triangle *b d e*. (Fig. 15.)

14. The two following examples were taken from the reports of the Woolwich examinations.

A hill slopes downwards towards the south west in such a manner that a line on its surface running north and south has an inclination of 30°, and a line on its surface running east and west, has an inclination of 10°. The hill falls suddenly into a horizontal plane at its base: show the angle which the intersection of the plane of the hill

side, with the horizontal plane, makes with the line running east and west; and show also on the plane of the hill side the position of a line inclined to the horizon at an angle of 20°, and the real profile angle of the plane. (Fig. 16.)

The line *a b* shows the direction east and west of the given line, *a c* its altitude, and *c b* its given inclination 10°.

The line *a d* shows the direction north and south, *a e* the altitude, and *e d* the slope of the second line. The position of *b d* shows where the hill falls in the horizontal plane.

The line *a k*, drawn perpendicular to *b d*, and containing the triangle *a k l*, shows the profile angle of the plane and the inclination of that plane to the horizontal plane.

The line *e f*, forming with the horizon an angle of 20°, and whose base *a f* is transferred in *a g*, shows the position, and the section *a g h* shows the inclination of the line contained by the plane.

15. A rectangular building 50 feet long, 25 feet wide, is to be covered by a hip roof whose height equals ¼ the width of the building, and the pitch or slope of each of its four surfaces is to be the same. Draw the plan of the roof, showing the ridge and hips. Determine also the inclination of the hips to the horizon, as well as their length.

The roof in plan forms the parallelogram *a b c d*, 50 feet long by 25 feet broad. A medial line, passing longitudinally through the centre determines the position of the ridge, whose length is determined by the breadth of the base of the slopes.

That breadth equals half that of the roof, or 12.5 feet all round. The pitch of the four surfaces being equal, the altitude of the ridge, which corresponds to ¼ of the breadth of the roof, will be 6.25; therefore the extremities of the ridge *e f* are 6¼ feet above the horizontal plane.

To obtain the slope and height of the hip, construct an elevation. Draw *e h* 6.25 feet perpendicular to *b e*, *b h* will show the angle of inclination and length of the hip rafter.

The right angle triangle *g f k* of the same altitude as *e h*, will show in the same manner the profile angle and pitch of the sloping surfaces. (Fig. 17).

Exercises.

1. Determine the length of a line whose scale equals 1.2 inches, but known to be inclined at an angle of 37° with the horizon.

2. Four horizontals are parallel and respectively .7, .2, and .4 inch distant from each other; their indices are 50, 25, 30, and 40 yards above the ground. Construct a profile showing the intersection of the planes they indicate. Scale, 40 to 1 inch.

3. Draw 7 lines each 6 inches long; the first two are .5 inch apart, the next .7, the next .9, and the last two each .7 inch apart. Let the indices of these lines be 0, 57, 63, 54, 28, 47, 93. Determine the intersection and scales of the given planes contained by those lines. Scale 40 units to 1 inch.

4. Draw 4 lines, each 6 inches long and .5 inch apart. Suppose those lines to represent the contours of a hill-side, at vertical intervals of 50 feet. Represent on the plan of the hill-side the position of a road inclined to the horizon at a slope of $\frac{1}{20}$. Scale, 200 to 1 inch.

5. A hill slopes at an angle of 10°. Show on it contours at 10 feet vertical intervals, and draw the plan of a road cutting the contours, and having an inclination of 1 in 30. Represent the position of the road by a single line.

6. Draw 6 parallel lines at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{2}$ inch apart.

Suppose these to represent contours of a hill side, at 10 feet vertical intervals, drawn on a scale of 400 to 1 inch. Show the central line of a road, which should have a regular inclination of $\frac{1}{30}$ throughout.

7. 6 lines seven inches long and .4 inches apart represent the contours of a hill-side, at vertical intervals of 5 feet drawn on a scale of $\frac{1}{1200}$: represent on plan, the portion of a road 28 feet wide, and having a slope of 1.25.

8. Draw 6 parallel lines 45 feet long at the respective distances of 6.5, 9, 1.5, 4.5 and 7 feet apart; join their extremities so as to form a parallelogram 45 feet long by 28.5 feet broad. Let the indices of these lines be 0, 6.5, 8, 3.5, 3.33 and 0, so as to represent the height of each line above the horizon. Construct two vertical sections of the solid, the first at right angles with the longest side, and the second forming an angle of 45° with the same.

Let both ends of the solid be so inclined as to form a slope of 1 in 4, *i.e.* a base of 1 foot to a rise of 4 feet, and draw horizontal contours to the solid at 1 foot vertical intervals. Scale, 10 feet to 1 inch.

9. Find by construction and figure the horizontal distance between the extremities of a straight line 570 yards long, supposing the extremities to be 18 and 87 feet respectively above the horizontal plane. Scale 100 yards to 1 inch.

10. A plane 4 inches long, inclined at an angle of 45°, represents the talus of a terreplein, its two horizontals represent a difference of altitude of 14 feet. Determine the position and length of a ramp that shall have a slope of $\frac{1}{30}$.

11. Show by its horizontal contours, .5 inch apart, a plane inclined at 65° to the horizon: on this plane draw a line 2 inches long, inclined at an angle of 35° to the

horizon, and another line 1.5 inch long also in the same plane, making an angle of 86° with the first line.

12. A hill side has an inclination of 33° to the horizon: ascertain by construction the difference of level between two points 100 yards apart measured on its slope.

13. A hill slopes at an angle giving 1 of height to 9 of base, and two points are marked on it in the direction of its greatest slope. Find by construction, on a scale of 50 feet to one inch the horizontal and vertical distances between those points.

14 Two pickets have their upper extremities on the same level 12 feet apart, and stand vertically on the side of a hill 7 and 11 feet respectively. Construct the scale of inclination of the hill, and give the fraction which it represents.

MISCELLANEOUS EXERCISES IN ORTHOGRAPHIC AND HORIZONTAL PROJECTIONS, DESCRIPTIVE GEOMETRY, ETC.

1. Determine by horizontal contours the projections of a sphere 4 inches in diameter, and of a cone 4 inches high; contours .25 inch apart.

2. Determine the length and inclination of a given finite straight line, and conversely draw a line of a proposed length and inclination through a given point.

3. The indices of two points 2.7 inches apart are 17 and 49: construct the scale and inclination of the line joining them.

4. Determine the horizontals and inclination of a plane containing three points, A 16, 2 inches distant from C 41 and 1.7 inches from B 37: B and C are .75 inch apart. Determine the slope of the plane, the scale of the plane, and the scale of slope of the plane. Scale 10 units to 1 inch.

5. Two horizontals parallel to each other represent respective heights of 9 and 27 feet above the ground: determine the position of an intermediate point 18 feet high.

6. Determine the inclination of a plane whose scale is a line 1.6 inches long, numbered at each of its extremities 5 and 22. Scale 10 units to 1 inch.

7. Determine the intersection of two planes whose bases form an angle of 85°, and whose slopes are respectively 36° and 52°. Scale 10 units to 1 inch.

8. Draw two planes inclined at angles of 48° and 63° respectively, intersecting in a line inclined at 39° with the horizon, and find the dihedral angle contained by them. Scale 10 units to 1 inch.

9. Through a line inclined at 38° to the horizon, draw a plane making an angle of 48° with another plane inclined at 23° to the horizon.

10. An equilateral triangle of 3 inches side rests on one of its angles, and has the sides adjoining that angle inclined to the horizon at angles of 25° and 35° respectively.

11. Draw a plane inclined at 57° to the horizon: in this plane place a straight line inclined at 43° to the horizon, and from any point in this line erect a perpendicular to the plane 1.7 inches long.

12. Determine the intersection of two planes inclined at 36° and 56° to the horizon, when the projections of their lowest horizontals are parallel and 1 inch distance from each other.

13. Construct the projections of a cube of 2.5 inches side, having two of the adjacent angles of its base inclined at angles of 31° and 42° to the horizon. Show the intersection with the cube of a horizontal plano .75 inch below its highest point.

14. Draw a line inclined at 38° with the horizon: through it draw a plane inclined at 55°, and in the latter place a line making an angle of 35° with the first line.

15. Draw a plane making an angle of 38° with another inclined at 46° to the horizon, and passing through a line (not in the given plane) inclined at 32° to the horizon.

16. The face of a cube whose edge equals 1.7 inches is inclined at 50° to the horizon: one of the diagonals of this face is inclined at 25°. Project the cube.

17. Draw a horizontal contour .25 inch vertically below the highest point of the cube, and make also an elevation of the cube on a vertical plane parallel to either of the diagonals.

18. The piers supporting the groined arches of a building are 10×6 feet in plan, 20 feet apart in the direction of their length, and 16 feet in the direction of their breadth. The larger arch is a semicircle, the smaller an ellipse whose semi-minor axis equals 5 feet. Draw the plan and a transverse section through the soffit of the smaller arches, and represent in plan the soffits of both arches by horizontal contours at 1 foot vertical intervals. Scale $\frac{1}{30}$.

19. A cube 3 inches edge has one of its faces forming an angle of 25° with the vertical line: another face is inclined at an angle of 38° with the horizon. The solid is perforated by three cylindrical holes 1.5 inches in diameter, whose axes cross each other in its centre. Show horizontal contours at .2 of an inch.

20. The plan of a line 4 inches long is 1.7 inches long: at what angle is the line inclined to the paper?

21. Six parallel lines, 8 inches long and .4 inch apart, represent the contours of a hill-side at vertical intervals of 5 feet, drawn on a scale of $\frac{1}{1200}$. Represent in plan on the hill-side the position of a road inclined to the horizon at a slope of $\frac{1}{30}$.

22. A hill slopes at an angle of 20°; show on it contours at 10 feet vertical intervals, and show the position of a road cutting the contours at an inclination of 1 in 4. Scale 100 feet to 1 inch.

23. A cylinder 5 feet in diameter is met by another cylinder 3 feet in diameter, at an angle of 45°, the lowest points of their curves are on the same level: show in plan the intersection of their surfaces. Scale 2 feet to 1 inch.

24. Find by construction, and figure very exactly, the horizontal distance between the extremities of a straight line 480 yards long, supposing these extremities to be respectively at 24 and 79 feet above the horizontal plane. Scale 100 to 1 inch.

25. The vertical projection of a line is represented by a line 2.7 inches long, inclined to the horizon at an angle of 33°; its horizontal projection is represented by a line 2.1 inches long: fine the real length of the line which will satisfy these conditions.

26. A sphere 4 inches in diameter is intersected by a cylinder 2.5 inches in diameter, whose axis passes at 1 inch from the centre of the sphere.

27. A right cone 4 inches diameter at its base and 5 inches high is intersected by another right cone 2.3 inches diameter and 4.5 inches high, the parallel distance between the two axes is 1 inch.

28. A right cone of the same dimensions is intersected by a cylinder 1.5 inches diameter, whose axis at 1 inch above the horizontal plane crosses that of the cone perpendicularly.

29. The axes of two cylinders whose diameters are respectively 2 and 3 inches, intersect at right angles: show their projection.

30. Show the development of a parallelopiped 2 inch edge, whose ends are squares of .5 inch side.

31. Show the development of a hexagonal pyramid 1 inch side and 4 inches high, obliquely truncated at $\frac{3}{7}$ of its height, and at an angle of 45° with the horizon.

32. Show the vertical projection of a triangular flag, whose pole, 10 feet high, is inclined at an angle of 50° to the horizon, and forms an angle of 70° with the vertical plane; the lower edge and that attached to the pole are respectively 4 feet long, and the upper edge is 5 feet. Scale .5 inch to 1 foot.

33. A hill has an inclination of 37° to the horizon: find by construction the difference of level between two points 120 yards apart, measured on their slope.

34. A hill slopes at an angle giving 1 of height to 7 of base, and two points 130 yards apart are measured in the direction of its greatest slope: find by construction, on a scale of 50 feet to 1 inch, the horizontal and vertical distances between the points.

35. Determine the horizontal projection and contours of a cone 4 inches high and 3 inches diameter, whose axis forms an angle of 75° with the horizon: contours at .25 inch.

ISOMETRIC DRAWING

Is a species of Orthographic projection (whose name derived from two Greek words, *Iso-metron,* signifying equal measurements) owes its origin to Professor Farish, of Cambridge, who introduced this method in order to represent on one plane, and according to certain rules and scales, with their three dimensions of length, breadth and thickness, objects which hitherto could only be described by means of plans, front and profile elevations, sections, and sectional elevations, and likewise to convey at the same time a kind of bird's-eye view of the general appearance of the object uninfluenced by the rules of perspective.

In order to obtain this result, let us imagine a cube, seen obliquely, so as to show three of its faces—one of its diagonals being perpendicular to the plane of projection. (Fig. 4 A.) In this position the square faces of the cube will naturally alter their shape, the three visible faces of the cube presenting together the shape of a regular hexagon having all its sides or edges equal, and equally inclined to that plane; each square face of the cube becoming, in fact, converted into a rhombus, all the angles of which change from 90° either to 60° or 120°.

The object in this method has the appearance of being cut out from the cube, each of the visible faces of which contain either the length, breadth, or height required.

Its practical application is useful in many cases, such as when the construction of certain pieces of architecture or machinery, builders or carpenters' work, and, in general, those objects whose principal parts lie at right angles to each other, have to be represented for purposes of elucidation. There is, however, a limit to its use, for should it be attempted to adapt it universally to intricate objects, the result would often produce ridiculous caricatures.

Construct a square isometrically. (Fig. 1, Pl. XLVIII.)

Let *a b c d* represent the face of a square which is to be projected isometrically.

Draw the two diagonals *a c*, *b d* at right angles to each other.

From *a*, with radius *a o*, describe the arc *o e*.

From *o*, with radius *o a*, cut the arc *o e* in *g*.

Bisect the angle *o a g* in *k*, and in the direction of *k* draw the line *a l*.

Draw *c m* parallel to *a l*, and join *a m*, *l c*.

The rhombus *a l m c* will be the isometric projection of the square *a b c d*, whose angles will be converted from right angles into *m a l* and *m c l*, angles of 60°, and *a l c*

and *c m a* into angles of 120°; the square, in assuming this position, appearing to revolve on the diagonal *a b* as upon an axis: although the length of that axis does not change, it is evident that the real sides of the square vary with the angle of inclination of the plane of which they are the boundaries. The sides *a l, l c, c m, m a* become the isometrical projections of the sides *a b, b c, c d* and *d a*. The angle *o a b* becomes converted from 45° to 30°: let the length of *a b* represent 2 inches, then *a l* will be the isometric representation of 2 inches, or about $\frac{9}{11}$, or more correctly .8164 of the original. The isometric axis *l m* assumes likewise the proportion of .5773 of its real length, as we will show.

But in practice, these lines bearing the same constant proportion to each other, and their different modes of construction being simple, it generally becomes unnecessary to obtain those dimensions by calculation; indeed, it is but seldom that any isometric scale is at all required, the adoption of the isometric lines divided according to any ratio being usually sufficient for all purposes.

Application.

Construct the isometrical scale of a line 4 inches long. (Fig. 2.)

Draw the line *a b* 4 inches long, and divide it according to the required denomination: make the angle *b a c* = 15°, and the angle *a b c* = 45°, the length *a c* divided proportionally to *a b* forms the isometric scale.

On the diagonal *a b c*, construct the isometric projection of the side *a d*, its scale, and the comparative scale of the small diagonal *b e* (part A). On part B show the isometric method of determining the graduations of an arc. (Fig. 5, Pl. XLIX.)

Let *a d e* represent half a square, and *a c* its diagonal bisected by the perpendicular *b d*; make the angle *b a e* = to 30°; divide *a d* into any number of equal parts (say 10) perpendiculars to *a b*, drawn from every point determined on *a d* and meeting *a e*, will show the isometric scale.

Divide *a b* into as many parts equal to those determined on *a d* as it will contain (about 7.2). From each of these parts, draw lines parallel to *a e*, meeting *b e*, and dividing it proportionally: *b e* will be the isometric axis scale required.

Now, if we consider half the minor axis *b e* as 1, we will find that the corresponding number of equal parts into which it may be divided (in this case about 7.2) equals 1.41421 of that length, and that the real or major semi-axis *a b* equals 1.73205 of *b c*, these three distances being in the ratios of $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$. Again, we have already mentioned that the real sides *a b*, *b c* (Fig. 4 A) become converted into the sides of a rhombus, *a b′ c*, whose diagonal remains uninfluenced and retains its original dimensions. Now, the square of *a b* + *b c* = *a c* (47.1), if we transfer that distance *a c* (Fig. 4 A) to *a′ b′* (Fig. 4 B), and make *b″ d* equal to *a b* and at right angles to *a′ b″*, the square of *a′ b″* + that of *b″ d* will equal the diagonal of the cube *a′ d*; if we take *a′ d* as 1 unit, then *a′ b″* will equal .8164 of that line, and *b″ d* .5773 of the same.

Let the fourth part of the square *b c d* contain the quadrant *f′ g*, whose circumference is divided into any number of equal parts (say 9): draw *c e* the isometric projection of *c d*: from the point *h*, in the bisection of *c d*, draw the perpendicular *h i*, determining the tangential point of the curve: the point *g* will be common to both curves. From *g* determine the point *k* at the extremity of a line parallel to *c e*, draw by hand the curve *g i k*: from every graduated point on the quadrant, draw perpendiculars to *b c*, meeting

the elliptical curve *g i k*. From centre *b* draw lines radiating to each of these points.

The curve *g i k* will be graduated in the same ratio as the quadrant *g h f*.

The following method of obtaining the scale is often adopted: draw *a b*, *a c* at right angles to each other, and of equal length; join *b c*, and transfer that distance from *a* to *d*; join *b d* and divide it according to required conditions, the lengths *b c* or *a d* will be the isometric projections of *b d*. (Fig. 3.)

The scales in Fig. 5 show the method of projecting isometrically irregular figures, and a careful inspection of Fig. 6. ought to make them evident without any further explanation.

To draw an isometric cube of 1.75 inches edge. (Fig. 7.)

On the isometric scale *a c* (Fig. 2.) take ¾ inch, and from centre *g*, with that distance as a radius, describe the circle *a c e*. From *a*, with the same radius, determine and draw the six chords *a b*, *b c*, *c d*, *d e*, *e f*, *f g*; join also *a g*, *c g*, *e g*, which will complete the projection of the cube required.

It is clear that should we require only the isometric projection of the upper face of the cube, the only operation required would be, on the line *g d* as a base, with a radius of 1.75 isometric inches, to construct the two equilateral triangles *g d c* and *g d e*, which combined would form the rhombus *e d e g*.

The isometric projection of a square being a rhombus, that of a circle inscribed in it will become an ellipse inscribed in the rhombus.

Let *a b c k* represent the isometric projection of the square *a b c d*, containing the circle *e f g h*. (Fig. 8.)

The diameter *m n* of the circle, when seen in the direc-

tion of the longest diagonal of the square, will remain common both to the square and to the rhombus.

The diameters of the circle *e g*, *f h* being parallel to the sides of the square *a b*, *b c*, draw *o p q r* parallel to *a k*, *k e*.

The points *o q*, *p r* will be to the sides of the rhombus as the points *e f g h* are to those of the square.

If from *m* we draw the chord *m s* parallel to *a b*, and from the same point *m* we draw *m t* parallel to *a l*, *m t* is to the rhombus as *m s* is to the square: the point *t* therefore becomes the projection of *s*, and the opposite point *u* is obtained in a similar manner by drawing *n u* parallel to *c k*.

With the hand, draw the elliptical curve through the points *m o t q n p a r*.

If from *k* as a centre, with radius *k l*, we describe the circle *k a v c*, the side or chord *a l* carried six times round the circumference *u w v x* will complete the cube.

The long diagonals *a v*, *v c* will bear to the short diagonals *w k*, *k x* the same proportion as *a c* bears to *l k*, and the ellipse inscribed in each rhombus will be obtained by following exactly the same method as that just described.

Fig. 9 includes and gives the solution of most of the difficulties met in isometric projections. It represents a hollow cube 1.5 inches edge whose sides are .2 of an inch in thickness; its upper or horizontal plane is surmounted by a hollow cylinder .5 inch high, .2 inch thick, and 3 inches in diameter. Its right hand plane contains a truncated cone .9 inch high, whose diameter at the base equals 1 inch and upper diameter equals .7 inch: the left hand plane is periorated by a circular hole 1.8 inch in diameter.

The cube is constructed according to isometric scale as explained in the last example, and the measurements ob-

tained from the isometric scale. The circles on the upper and left hand planes are obtained in the same manner; the only natural or true dimensions being taken for the diameters of the circles on the longest diagonals: all the other dimensions are obtained from the isometric scale.

To construct the cylinder on the upper plane.

From the points *a b c d e f g h,* determining the projections of its base, erect perpendiculars .5 isometrical inch high, and through their extremity, 1, 2, 3, 4, 5, 6, 7, 8, describe an ellipse. On the line 1—5 take true distances of .2 inch for the thickness of the cylinder and determine them at the points 9, 10. Draw the diagonals 2-6 and 4-8, on which find the isometric distance of .2 inch; the chords 9-11 and 10-12 will determine the thickness of the cylinder at these points: the circle is to be drawn as before.

From the points 9, 10, 11, 13 and 14 drop perpendiculars. .7 isometric inch long, and through these draw an arc parallel to the upper inner one, which will determine the inner height of the cylinder.

For the circle on the left hand face, the thickness of the board or edge of the cube is attained in a similar way by drawing lines from *k l m n o* parallel to the short axis, and projecting on the isometric distances of .2 of an inch.

As the base of the truncated cone on the right hand plane is 1 inch in diameter, a rhombus of that dimension, drawn by measuring from centre A to B and C half that size, and drawing lines parallel to the sides of the original plane; will contain the ellipse forming the isometric base of the cone, whose height is found by measuring .9 isometric inch frem A to F, which latter point in its turn becomes the centre of a smaller rhombus, whose dimensions are obtained in the same manner, and contains the ellipse forming the extremity of the frustum of the cone,

and whose thickness is obtained as in the case of a cylinder: lines drawn tangent to these curves form the sides and complete the figure.

A square piece of ground is surrounded by four walls 2.5 feet thick and 16 feet high; each wall is perforated by a semicircular arch 12 feet in diameter, springing at 8 feet from the ground; one half of the two side arches has been taken down, leaving only their piers standing; the rear arch and piers are completely demolished: construct their isometrical projections by a scale of 10 feet to 1 inch. (Fig. 10.)

It is supposed that the proper comprehension of the foregoing examples will ensure the correct solution of the following questions; mostly founded on similar ones given in examination papers.

1. In what position are objects supposed to be viewed in isometric projection? For what purposes is it especially adapted, and what is the proportion between a real line and its isometric projections?

What is the meaning of the term isometric, and who introduced it in England?

2. Construct the isometric projection of a line 6 inches long.

3. Give that of a square containing a circle 4 inches in diameter.

4. Give the isometric projection of a cube of 2 inches edge, and show the scale.

5. Give that of a rectangular tray, 4 inches long, 3 inches wide, 1.2 inches high, with sides of .3 of an inch in thickness.

6. That of a cylindrical box made of $\frac{1}{2}$ inch deal, whose exterior diameter equals 10 inches, and height 2 inches. Scale, $\frac{1}{2}$ full size.

7. That of an arched postern, 9 feet wide, 12 feet to

the crown of the arch, which is semi-circular, and cut through a wall 5 feet thick and 16 feet high, showing a length of wall of 5 feet on each side of the postern. Scale $\frac{1}{30}$, or 3 feet to 1 inch.

8. Of a box 3 feet long, 2 feet wide, and 1.6 deep, made of $\frac{1}{2}$ inch board, with a circular hole 6 inches in diameter on each side and end. Scale $\frac{1}{12}$.

9. A table 3 feet wide, $4\frac{1}{2}$ feet long, 3 feet high, the top is 2 inches thick. The legs 2 inches square, are fixed at 2 inches from the outside of the table. A circular hole $1\frac{1}{2}$ foot in diameter, is in the centre of the top of the table.

10. A truncated cone, base 3 inches in diameter, height 4 inches, diameter of upper end 2 inches.

11. A piece of timber 4 feet long, 2 feet wide, and 3 inches thick, a hole in the shape of a frustrum of a cone is bored through its thickness, in the centre of its length and breadth. The upper diameter of the hole is 1 foot, the lower diameter 6 inches. Scale $\frac{1}{8}$.

12. A hexagonal right prism 9 feet long, each edge of the base measuring 2 feet, and having one of its faces resting on the ground. Scale $\frac{1}{24}$.

13. A box 6 feet long, 4 feet wide, and 3 feet high, lid 6 inches thick, opened at an angle of 40°. The sides of the box 2 inches thick. Scale $\frac{1}{12}$.

14. A one roomed cottage, of which half the roof has been removed. Exterior dimensions, length 16 feet, breadth 14 feet, height of walls 12 feet, of roof 5 feet, wall 1 foot thick. In the short side, a door 3 feet wide and 8 feet high, is ascended with 2 steps, each 1 foot broad and 4 feet long; in the long side are two semicircular windows, each 5 feet 6 high to the springing of the arch, and 3 ft. 6 inches wide, whose sills are at 2 feet 6 inches from the ground. Scale 4 ft. to 1 inch.

15. Of an octagonal prism of 1 inch side, 5 inches high, standing on one of its ends.

16. Of a hexagonal right prism of 1 inch side and 5 inches high, standing on one of its ends, and surmounted by a hexagonal pyramid 3 inches high, and whose edge coincides with the edges of the prism.

17. Of a flight of 3 steps, each 10 feet long, 1.6 wide, and 9 inches rise. Scale $\frac{1}{24}$.

18. Of a small case of instruments with a lid open at an angle of 48°.

19. Of a double cross standing on a pedestal, choosing your own dimensions

20. Of a block of wood 4 inches long, 2.5 inches broad, and 2 inches high, having one of its upper corners cut off in such a manner that the section made shows an equilateral triangle of 1 inch side.

PERSPECTIVE.

The term "Perspective" is derived from the Latin "Perspicere," to see through: its object is to represent on a plane the exact form or representation of objects as they would appear were they seen through some assumed transparent medium placed between them and the spectator.

In the Diagram 1, Pl. LI, I have endeavoured to give some idea of the meaning of the terms used by artists, &c. to indicate the position and names of the several points, lines, planes, &c. used in perspective.

Let A represent the ground, or horizontal plane.

The ground, or horizontal plane, is the horizontal surface upon which the object and spectator are supposed to stand.

Let B represent the picture plane.

The picture plane, or perspective plane, is any supposed transparent medium interposing between the spectator and the object, the lower part of which, G L, meeting the ground plane, to which it is perpendicular, is termed the ground line. N.B.—These two planes are assumed to be of indefinite extent.

Let C represent the original object. An object is given at first in plan, which determines its ***position, length and breadth,*** according to actual dimensions, but not its height.

Let E represent the eye of the spectator, placed vertically at a given height above its position, on the ground plane S, termed the station.

The line E-PS, ***always*** perpendicular to the picture plane, is termed the line of sight, axis of vision, or line of direction; it proceeds from the eye of the spectator to the picture plane, at which extremity it is termed point of sight: the point of sight, P S, or perspective centre, is therefore the further extremity of the line of sight.

H H, the horizon, or horizontal line, is a line drawn on the picture plane, parallel to the ground line, and of indefinite extent, perpendicular to the line of sight.

Diagram 2 gives the actual plan of the object to be put in perspective. V S V, the angle of vision, or visual angle, is that space which the spectator can embrace with his eye without turning his head to the right or left. It forms a sector of 60°, so that the line of sight, L S, becomes in plan a perpendicular, bisecting an equilateral triangle, whose base lies on the horizon, and whose apex coincides with the eye of the spectator, S.

P D, points of distance, are points placed right and left of the point of sight, equal to its distance from the eye.

L C, line of contact, is a line drawn in the prolongation of one of the sides of the object to the picture plane. Its

use is to determine in the elevation the height of the object and of its several parts. N.B.—When parallel lines or planes of different altitudes are receding from each other, it is often convenient to project several lines of contact.

V R, visual rays, form a pyramid, or cone of rays proceeding from every visible point or angle of the plan, and converging towards the eye of the spectator till it crosses the picture plane, on which it forms. P, the picture, or representation of the object as seen in Diagram 3, which represents the elevation of the picture plane, whose base, or ground line, G L, is supposed to be removed from its original position, H H, Diagram 2, and reduced to the horizon. The vertical projectors of the two diagrams coinciding with each other.

Vanishing lines, V L, are lines proceeding from the eye of the spectator to the horizon, parallel to the sides of the object. Their extremities, or points, where they meet the horizontal, are termed vanishing points. V P.

N.B.—All lines, or planes which are parallel to each other in the plan, tend or converge towards the same vanishing point in the elevation.

We have already observed that the axis of vision is always represented by a line perpendicular to the picture plane, and proceeding from the eye of the spectator. It follows, therefore, that not only its length is equal to the distance of the eye to the picture plane, but that likewise its height above the station determines that of the horizon above the ground line, to which it is parallel in the picture plane.

It is perhaps unnecessary to observe that the height of the horizon varies in every case with that of the eye of the spectator. Whatever may be the position of the latter, whether he is lying on the ground, or standing up; whether he is placed on the top of a ladder, a tower, or a mountain,

the apparent height of the horizon always equals that of the eye above the ground line.

When the sides of an object form any angle with the picture plane we use the method of "Oblique Perspective."

When the sides of an object are perpendicular or parallel to the picture plane, we employ that of "Parallel Perspective."

The method of putting an object in parallel perspective is simple. We have already said that vanishing lines are lines drawn from the eye of the spectator to the horizon parallel to the sides of the object. It is evident that this applies only to oblique perspective, for were we to attempt by this method to put in perspective a line parallel to the picture plane, we would find it impossible. We must, therefore, adopt some other means. The most convenient is to suppose the object to be enclosed in a square, each of whose diagonals forming an angle of 45° with the picture plane, and whose sides being perpendicular to the same, will converge towards the point of sight, which in this method becomes the sole vanishing point.

The following diagrams will, I trust, prove sufficiently explicit.

Required the perspective of a square in three positions, A, B and C.

In the first position A, the point of sight, P S, is opposite the square, and contiguous to the ground line, G L.

In the second position B, the square is still contiguous to G L, but to the left of P S.

In the third position C, the square is at a given distance from G L, and to the right of P. S. (Fig. 3.)

Method of Construction.

1. Draw indefinitely G L, the ground line.

2. Parallel to G L, and at the assumed, or given height of the eye, draw indefinitely H H for the horizon.

3. In any convenient position on H H determine the point of sight, P S.

4. On the right and left of P S determine the points of distance, P D, equal to the assumed or given distance from the eye of the spectator to the picture plane.

5. Construct the squares A and B according to given dimensions, and contiguous to G L. Also the square C, at the given distance from G L, and according to given conditions produce its sides $a\ a'$, $b\ b''$ to meet G L in $a''\ b''$.

6. Draw the diagonals $a\ b$ of the squares A and B, meeting the ground line G L, and draw the diagonal $b\ a'$ of the square C, meeting G L in c'.

7. From $a\ b$, $a\ b$, $a''\ b''$ draw the vanishing lines converging to P S, which is the common vanishing point to all the lines perpendicular to the picture plane.

8. From the points, $b\ b$ and c' draw the oblique lines to P D. The intersection of those lines in $d\ d\ d$ and f by a parallel to G L, will determine the perspective of the squares.

This is the foundation of all "Parallel Perspective."

In practice it is not even necessary to construct the squares, for knowing that the lines $a\ b$ are diagonal to the squares, they do not affect the position of $b\ d$ in G L B with respect to their prolongation to P D. Similarly in C the distance $a''\ c'$ is equal to $a''\ a'$, determined in the picture by the distance between the parallel $a''\ b''$ and f and g.

The preceding being well understood, the learner will

find little difficulty in constructing the following figures, all of which are based on the same principles.

Construct the perspective of three squares, A, B, and C.

A contains a square whose sides are oblique to its own.

B contains a regular octagon, and

C contains a circle.

Like in the preceding case, determine again at will the point of sight, P S, and the two points of distance, P D.

The perspective of A requires no explanation.

For that of B, draw the perpendiculars *c d e* parallel to *a b*, and project them to P S. They will be cut in 1 2 3 by the diagonal *c*-P D, which will determine the position of the horizontals *f g h* projected, forming a series of squares, through which will be determined the corresponding sides of the octagon.

In the perspective of the circle a few words may be necessary.

Draw the two diameters of the circle and the two diagonals of the square enclosing it, *e f g h*. The intersection of these with the circumference *a b c d* will determine the points *k l m n*. Inspection of the diagram will show that the perspective is obtained in the way already explained.

Figs. 6 and 7 show the mode of placing tesselated pavements of various kinds in perspective.

Required the perspective of a stick 6 feet high, 7 feet distance to the left of the point of sight, and 3 feet from the picture plane. Height of the eye 4 feet. Distance from station to point of sight 8 feet.

Also the perspective position of a bird flying at 7 feet above the ground, 4 feet from the picture plane, and 7.75 feet to the right of the point of sight. (Fig. 5.)

Draw the ground line G E, and the horizon parallel and 4 feet above it.

Determine P S, from which draw the perpendicular P S-O.

From O determine to the left, at 7 feet from it, A, for the foot of the stick: draw A B, on which determine its height (6 feet). At 3 feet to the right of A determine the point C, to determine the distance of the stick from the picture plane. To the right and left of P S determine P D at 8 feet (distance of the station to the picture planes). The lines converging from A and B to P S will be the vanishing lines of the extremities of the stick, whose position will be determined in *d* and *e* by the intersection of the line C drawn towards P D.

The second part of the question is precisely analogous to what was just explained.

Pl. LV. shows the method of putting solids in perspective.

Required the perspective of a cube .75 inch edge, one of whose faces is parallel to the picture plane and ½ an inch distant from it. (Fig. 8.)

The plan and perspective of the base are determined in the usual manner. The height is obtained by producing A B through G L and determining it on C D. Inspection of Pl. 54 will not only show how to complete the diagram, but also its analogy with Figs. 9, 10, 11, which represent a cube, a pyramid, a cylinder surmounting a square base, and a double cross.

The representation of objects in parallel perspective may often be simplified by omitting the plan altogether.

For instance, required the perspective of a cube containing on each of its faces a circle tangent to its side.

Draw G L and H H at the required height, and on it determine P S and P D: on G L take *a b* the breadth of the cube upon which construct the square *a b c d* containing the circle.

From *b* draw the diagonal *b*-P D, cutting *a*-P S in G.

On G L determine the points *b h i k g* distant to each

other in the same ratio as those determined on *a b* in *l m n*. From each of these points, draw lines converging to P D and intersecting *b*-P S in 1 2 3 4. Perpendiculars drawn from each of these points, and the inspection of the diagram will direct how to complete the figure. (Fig. 12.)

A portion of masonry containing in its frontage a semicircular arch and in its two sides the half of semicircular arches, is constructed after the same method, and should not need any further explanation. (Fig. 13.)

Practice.

A cube 2 inches edge and .5 inch from the picture plane has one of its faces parallel to it: put it in perspective.

The eye is directed to a point at 4 inches to the right of the nearer side, and 3 inches above the ground. The spectator is 6 inches distance from the picture plane.

Draw a cylinder 4 inches high and 2 inches in diameter under the same conditions.

Oblique Perspective.

When the faces of an object form any angle with the picture plane, we use the method of oblique perspective.

As several methods exist of placing an object in perspective, I have chosen that which speaks most to the eye, and which I have found to be most easily understood.

The adoption of the few following rules will be found sufficient to represent any object:—

1. Draw the picture plane.
2. Draw the line of sight in any convenient position, and at right angles to it.
3. Determine the distance from the picture plane to the object, and the position of the latter relatively to the line of sight.

4. Determine the distance from the picture plane to the station.

5. Determine the position of the nearest side of the object forming the smallest angle with the picture plane, and on that line construct the plan according to scale.

6. From the station, draw lines parallel to the nearest sides of the object and directed towards the picture plane: these lines are termed the "vanishing lines."

7. For a line of contact, produce the line forming the least angle with the picture plane till it meets it.

8. From every visible point of the object in plan draw the visual rays converging towards the station till they are met by the picture plane.

9. From every one of these points thus determined on the picture plane, draw below it and to any distance lines at right angles to it, for the projectors; these lines will determine the breadth of the various parts of the object as seen on the elevation: For the elevation,

10. At any convenient distance below the picture plane draw a horizontal for the ground line. N.B.—This ground line forms the base of the picture plane transferred from the plan and then supposed to be reduced to the horizon, so as to show its general appearance.

11. Above the ground line and parallel to it at the given height of the eye, draw the horizon, on which,

12. By means of their projectors on the plan, determine the position of the vanishing points.

13. On the projectors of the lines of contact, determine by scale the height of the various parts of the object.

14. Lines drawn from these points of height to the vanishing points of the respective planes or lines to which they refer, and meeting the projectors of the visual rays, will complete the picture.

No matter under what shape the question may be enun-

ciated, it may always be easily solved provided these simple rules are adhered to; the pupil is therefore recommended to understand them thoroughly. As the following examples are progressive, as well as the exercises, which are intended as applications to these rules, the solution of each question in its proper order will be found perfectly easy, and require at most a few verbal hints.

In order to facilitate the execution of these examples, numbers, showing the numerical order in which each line is to be drawn, and referring to the directions given above, have been attached to some of the diagrams.

For the conditions of distance of the picture plane to the station or object, for the angles which the latter make with the picture plane, the length or height of its several parts, &c., the learner must be guided by the enunciation, and not by the diagrams, which are mostly intended only to elucidate the subjects, and only represent the general appearance of the objects, without being drawn to any particular scale.

Find the perspective of a line 2 inches long, forming an angle of 30° with the picture plane. The eye of the spectator is supposed to be directed to a point at 8 inches distance from it, and 4 inches above the ground, but at 6 inches from the picture plane. (Fig. 14.) Scale of $\frac{1}{1}$.

To avoid repetition, and when not otherwise stated, we will suppose the above conditions with respect to the length of line or side to be 2 inches, the angle of the side of the object with the picture plane 30°, the distance of the eye to the picture plane 6 inches, and the height of the eye or horizon 4 inches.

2. Show the perspective of two lines at right angles to each other. (Fig. 15.)

3. Show the perspective of a third line forming a tri-

angle with the two already given in the previous question. (Fig. 15.)

4. Show the perspective of a square, and of its two diagonals. (Fig. 16.)

5. Show the perspective of a square containing a circle tangent to its sides. (Fig. 17.)

6. Show the perspective of a regular pentagon. (Fig. 18.)

7. Show the perspective of a regular hexagon. (Fig. 19.)

8. Show the perspective of three octagons joined together. (Fig. 20.)

9. Four squares of 1 inch side are placed with their points touching the centre of the sides of another square of the same dimensions, with its sides not parallel to the picture plane. Give its perspective view when the eye is 7 feet above and 14 feet distant from the centre, and when the rays from the nearest point to the eye make an angle of 115° with the side of the centre square. (Fig. 21.) Scale $\frac{1}{2}$.

PRACTICE.

10. Three hexagons of 2 feet side, placed in the form of a triangle, the side of which makes an angle of 20° with the picture plane, and 20 feet from it. The eye is supposed to be at 8 feet from the nearest point, and 6 feet above the ground.

11. A cube of two inches edge, forming with the picture plane an angle of 36°, and a right pyramid 2 inches high whose base forms an equilateral triangle of 3 inches side, one of which forms an angle of 35° with the picture plane, are 2 inches distance from each other. The eye of the spectator is directed to a point between the two solids. The other conditions are optional. (Fig. 22.)

12. Required the perspective of a seven-leaved screen, 6 feet high and 3 feet wide, placed in such a position that

no two leaves are in the same direction; other conditions optional. (Fig. 23.)

13. A flight of three steps, each 5 feet long, 1 foot wide, and 1 foot rise. Their front forms an angle of 30° with the picture plane, from which the observer's eye is 5 feet distant. Distance to object 6 feet; height of the eye 4 feet. Scale $\frac{1}{48}$. (Fig. 24.)

14. A hexagonal right prism 8 feet long, each edge of the base 2 inches wide, rests with one face on the horizontal plane. Draw its perspective. The picture plane is 12 feet from the nearest point, and makes with it an angle of 40°. Height of the eye 6 inches. Distance to object 7 feet. Scale $\frac{1}{4}$. (Fig. 25.)

15. A box 4 feet long, 3 feet wide, and 2 feet deep, has its nearest corner at 1 foot from the picture plane, which forms with the longest side an angle of 40°. The lid is open at an angle of 60°, and the eye of the spectator is 3 feet above the ground. Choose the other conditions to place it in perspective. Scale $\frac{1}{48}$. (Fig. 26.)

16. A square block of masonry, 20 feet high, 20 feet long, and 10 feet deep, is perforated by a semicircular arch having a span of 14 feet, springing at 8 feet above the ground: show its perspective, the longest side forming an angle of 35° with the picture plane. Distance from station to object 30 feet, to picture plane 28 feet; height of line of sight 8 feet. Scale 5 feet to 1 inch. (Fig. 27.)

17. Give the perspective of the double cross whose dimensions are indicated in the diagram.

Angle of the side of the cross with the picture plane 44°.

Object to station 14 feet, to picture plane 2 feet; height of eye 7 feet. Scale $\frac{1}{48}$. (Fig. 28.)

18. The perspective of a door-frame 7.6 high and 4 feet wide interior dimensions, and of a four-panneled door

partly open. The door-frame is parallel to the picture plane, and the line of sight is at $\frac{2}{3}$ of the height of the door.

The other conditions are optional. Scale $\frac{1}{25}$. (Fig. 29.)

19. Each of the faces of a cube of 4 inches edge contains a circle tangent to its sides. Its nearest face forms an angle of 30° with the picture plane. The eye is directed to a point 4 feet above the upper face of the cube: other conditions optional. Scale $\frac{1}{25}$. (Fig. 30.)

20. An octagonal pillar, 8 feet high and 1 foot wide, stands on a square base of 3 feet side and 1 foot high; 4 sides of the pillar are parallel to the sides of the base.

Horizon 5 feet; object to picture plane 2 feet; to station 10 feet: one of the faces forms an angle of 45° with the picture plane. Scale $\frac{1}{24}$. (Fig. 31.)

21. A block of masonry, 100 feet long, 20 feet wide, and 40 feet high, represents a portion of a bridge containing two semicircular arches of 30 feet span, springing from piers at 20 feet above the ground. The two extreme abutments are only 10 feet wide; the central one is 20 feet; roadway 18 feet wide; height of parapet 5 feet; angle with picture plane 30°; station to object 120 feet; to picture plane 100 feet. Scale 30 feet to 1 inch. (Fig. 32.)

22. A wash-hand-stand 3 feet long, 2 feet wide, and 4 feet high, with a circular hole in the centre of 1.6 diameter. The other details of construction are to be assumed. Angle with picture plane 28°. Station to object 7 feet; to picture plane 5 feet; horizon 6 feet. Scale $\frac{1}{24}$. (Fig. 33.)

The irregular position of the objects required to be put in perspective, often compels us to use the method of projections, in order to obtain their plan and to fulfil their other conditions.

23. A point .4 inch from the picture plane, and a second point at 1.3 inch distant from it and 2.1 inches distant

from the first point, represent the plan of a pole 3.2 inches long, of which the first point represents the foot: give its perspective, the other conditions being optional.

Let *a* represent the first point, *b* the second; the line *a b* will show the plan and direction of the pole. To obtain the height we must make an elevation, therefore draw *b c* perpendicular to *a b*, and from *a* as a centre, with a radius of 3.2 inches, equal to the length of the pole, describe an arc cutting *b c* in *d*: *b d* will represent the height of the upper extremity of the pole above the ground; the remaining part of the construction should require no further explanation. (Fig. 34.)

24. Show the perspective of a triangular flag, 4 feet long on one of its flying edges, 3 feet on the other, and measuring 3 feet long on the edge by which it is attached to the pole from which it is suspended. It is so placed that the pole, 8 feet long, is inclined to the horizon at an angle of 30°, and the plan of the pole is inclined to the picture plane at an angle of 48°: choose the other conditions as you prefer. Scale ½ inch to 1 foot.

This question being somewhat analogous to the one just explained, needs no further directions; I will merely observe that the plane *a b c d*, containing the elevation of the flag, may be represented in perspective by the plane *a′ b′ c′ d′*, of which the flagstaff is the diagonal, and from which the other parts may equally be easily obtained. (Fig. 35.)

25. An isosceles triangle has two of its sides equal to 1.9 inch, and the third side equal to 2.5. It stands on the horizontal plane with one of its angular points in the picture plane. Its longest side inclined to the horizon at an angle of 25°; its plane inclined to the horizontal plane at an angle of 68°, and the trace of its plane inclined to the picture plane at an angle of 20°.

The eye of the observer is 4.5 inches from the picture plane, opposite the point of the triangle which rests on the horizontal plane, and 2.5 inches above the horizontal plane. (Fig. 36.)

26. A right pyramid has an equilateral triangle of 2.2 inches side for its base, and is 3.8 inches high; it is laid on one of its faces: show its projection (1). Show likewise the plan and elevation of the same pyramid lying on one of its faces, with one side of its base forming an angle of 70° with the vertical plane (2). Draw the perspective of the same pyramid; horizon 2 inches, distance from the station to the nearest point of the object which coincides with the picture plane, 6 inches (3). Scale ½. (Fig. 36, 2.)

27. An octohedron formed by two square pyramids joined at their bases, each pyramid is 3 inches high and 1 inch edge, and is laid on one of its faces: its axis forms an angle of 30° with the picture plane; station to object 12 inches, to picture plane 9 inches, horizon 4 inches. (Fig. 37.)

28. One of the diagonals of a cube of 2 inches edge passes through the eye and is perpendicular to the picture plane: draw its perspective, other conditions optional. (Fig. 38.)

It is hoped that after having solved the preceding problems, the pupil will no longer require either explanation or demonstration, but will be enabled to solve any further questions.

When not otherwise given, the conditions are optional.

Required the perspective of a cube which has one of its faces horizontal, and the diameter of that face perpendicular to the picture plane: assume two positions of the eye, one above and the other below the upper surface of the cube.

The perspective of a regular tetrahedron 3 inch edge, having one of its faces parallel to the picture plane.

A block of stone 6 feet long, 4 feet wide, and 3 feet thick is placed on one end, and another of the same size is laid flat on the top of it: draw its perspective; station to object 9 feet, to picture plane 6 feet, horizon 6 feet, angle 40°. Scale $\frac{1}{24}$.

A table 4 feet long, 2.5 feet wide, and 2.5 feet high; the top is 2 inches thick, the legs 2 inches square and fixed at 1.5 inch from the outside of the table; a circular hole 1.5 foot in diameter is in the centre of the top of the table; the eye is 6.5 feet above the horizontal plane and 6 feet from the picture plane, which is 1 foot distant from the nearest part of the table, and at an angle of 40° with its longest side. Scale $\frac{1}{12}$.

The perspective of a cross, of which the upright is 5 feet high, cross piece 3 feet long and 1 foot below the top, each piece being eight inches square; the eye of the observer is 10 feet distant, 3 feet above the top, and in a direction making an angle of 45° with the front of the cross. Scale $\frac{1}{24}$.

A box 6 feet long, 4 feet wide, 4 feet high, lid 1 foot thick opened at an angle of 45°; horizon 5 feet, object to picture 6 feet, to station 10 feet; its longest side forms an angle of 30° with the picture plane: show details, thickness, &c.

A regular parallelopipedon stands on a horizontal plane with its longest side inclined to the picture plane at an angle of 20°.

Length of the solid 12 feet, breadth 10 feet, height 7 feet, height of eye 12 feet; line of sight directed to a point 2.5 feet to the right of the prolongation of the nearest perpendicular edge, station to object 20 feet. Scale $\frac{1}{70}$.

The perspective of a block of masonry is required.

It is 20 feet high, with a frontage of 17 feet and depth of 13 feet, and is perforated longitudinally and transversely by semicircular arches springing at the same height, viz., 10.5, and intersecting each other, the piers are 4.25 square; angle 20° with picture plane; line of sight 5 feet to the right of the nearest angle, horizon 6 feet, station to object 25 feet, to picture plane 20 feet.

The perspective of a rectangular block of masonry 24 feet long, 20 feet high, and 16 feet broad, pierced by an arch springing at a height of 10 feet, and of a semicircular form, with a span of 12 feet; the station is opposite a point on one side of its centre. Scale $\frac{1}{72}$.

The perspective of a right pyramid 3 inches high on a square base of 1.5 inch, one of whose sides forms an angle of 20 degrees with the picture plane, and whose nearest angle is 1 inch beyond it; the eye is directed to a point to the left of the pyramid: other conditions optional.

A rectangular prism 2 inches high, having for its base a triangle whose sides equal 2, 2.5 and 1.7 respectively, one of whose sides forms an angle of 30° with the picture plane, the nearest angle at 1 inch from it; the observer's eye is 6 inches from the picture plane, opposite the nearest angle of the base and 2 inches above the horizontal plane on which it stands.

One of the diagonals of a cube of 2 inches edge passes through the eye, and is perpendicular to the picture plane: draw its perspective.

The sides of a triangle equal 2.5, 2.5 and 2.75 inches respectively, the longest side is to stand vertical at a distance of .25 inch from the picture plane, the eye of the observer is to be 1.75 inch above the horizontal plane, the point of sight directed at about 3 inches on either side of the triangle, whose plane is to have an inclination of 30° to the picture plane: other conditions optional.

A cottage, of which half the roof has been removed, consists of one room, the external dimensions of which are, length 15 feet, breadth 12 feet, height 10 feet 5 inches; in one of the longest sides, which forms an angle of 30° with the picture plane, are two windows with semicircular heads, each 5.5 feet high to the springing of the arches, and 3.5 feet wide; 2.5 inches from the ground: in one of the shortest sides is a doorway 3 feet wide and 7.5 feet high, reached by two steps each 3 feet long, 1 foot wide, and 6 inches thick; ridge of roof 18 feet above the ground; the walls are 1 foot thick; the walls containing the windows and door are to be the nearest to the picture plane, which is 6 feet distant from it: station to object 20 feet, horizon 5 feet. Scale $\frac{1}{48}$.

The perspective of an equilateral triangular board, 3 inches side, one side resting on a horizontal plane and forming an angle of 40° with the picture plane, which it does not touch. The face of the board is inclined to the horizon at an angle of 55°; height of observer's eye 1.5 inch, point of sight 1 inch to the left of the object.

The perspective of an isosceles triangle whose base equals 2.9 inches and sides 2.1 inches. The triangle stands on a horizontal plane with one of its angular points in the picture plane; its longest side is inclined to the horizon at an angle of 20°, its plane inclined to the horizontal plane at an angle of 50°, and the trace of its plane inclined to the picture plane at an angle of 20°.

The eye of the observer is 4.5 inches from the picture plane, opposite the point of the triangle which rests on the horizontal plane, and 2.7 inches above the horizontal plane.

THE END.

G. NORMAN, PRINTER, MAIDEN LANE, COVENT GARDEN.

ERRATA.

Page 4, line 27, *instead of* "area" *read* "circumference."
„ 5, line 1 and 2, *instead of* "triangle," *read* "angle."
„ 7, line 20, *instead of* "$\frac{136}{60}$" *read* "$\frac{130}{60}$."
„ 13, line 5, *instead of* "9·" *read* "9.9 inches."
„ 14, line 29, *instead of* "Nudecagon," *read* "Undecagon."
„ 33, line 30, *instead of* "TV draw," *read* "TV, draw."
„ 40, line 7, *instead of* "AB in a line C," *read* "B in a line AC."
„ 40, line 25, *instead of* "contining," *read* "containing."
„ 41, line 5, *instead of* "61.30 and BXC=43.45°," *read* "61° 30 and BXC=43° 45′.
„ 42, last line, *instead of* "400 to 1 inch," *read* "400 yards to 1 inch."
„ 54, last line, *instead of* "mile," *read* "inch."
„ 55, line 21, *instead of* "4" *read* "40."
„ 65, line 22, *instead of* "5.1 inch" *read* "5.1 inches."
„ 79, line 17, *instead of* "*ce* and *df*," *read* "*de* and *cf*."
„ 83, line 19, *instead of* "contrast," *read* "construct."
„ 87, line 25, *instead of* "14° respectively to the," *read* "14° to the."
„ 88, line 8, *instead of* "*acd*," *read* "*ac*."
„ 93, line 33, *instead of* "22″" *read* "2′ 2′."
„ 94, line 27, *instead of* "sections," *read* "sectors."
„ 94, line 30, *instead of* "sectional," *read* "sectorial."
„ 103, line 4, *instead of* "adoption," *read* "adaption."
„ 111, line 20, *instead of* "1.5″ inch," *read* "1.5 inches high."
„ 112, line 12, *instead of* "vortex," *read* "vertex."
„ 121, line 12, *instead of* "auy," *read* "any."
„ 128, line 20, *instead of* "plane," *read* "plan."
„ 147, line 16, *instead of* "$\frac{3}{4}$" *read* "$1\frac{3}{4}$."
„ 149, line 31, *instead of* "frem," *read* "from."

FIG. 7

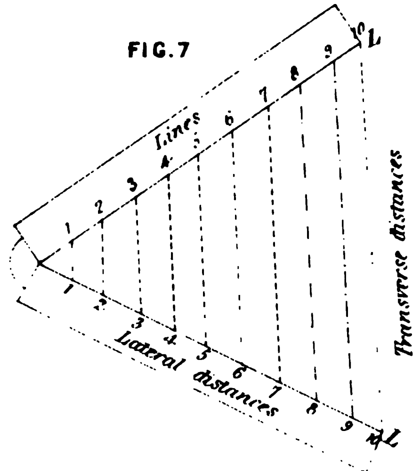


FIG. 8

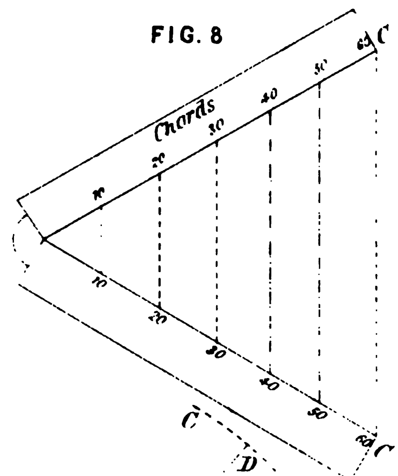
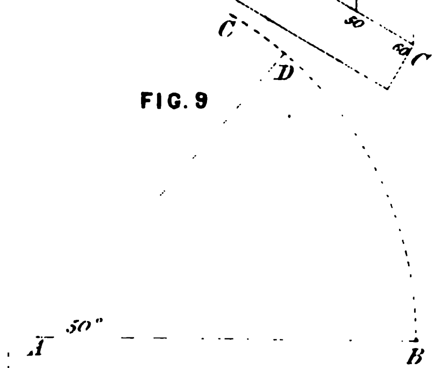


FIG. 9



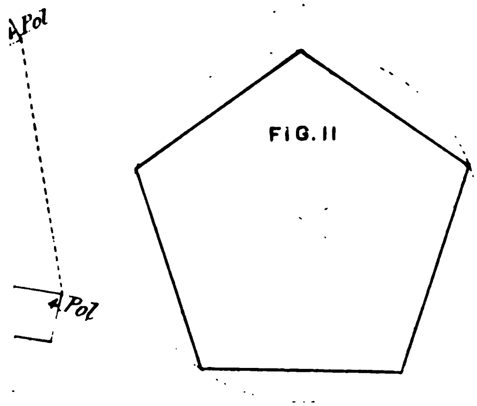


FIG. 11

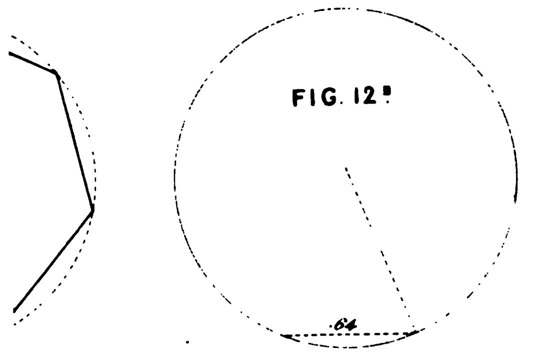
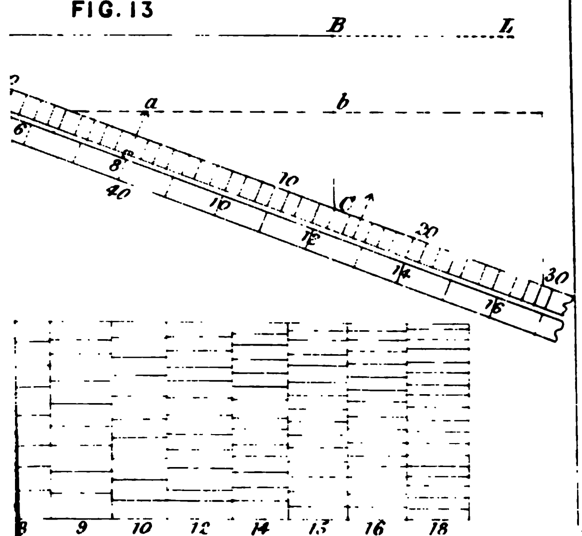


FIG. 12ª

FIG. 13



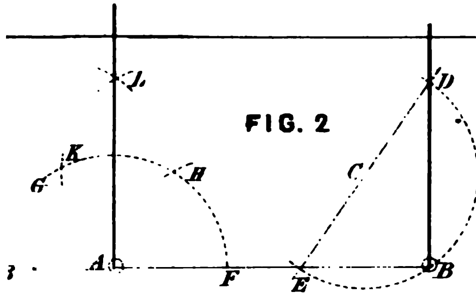


FIG. 2

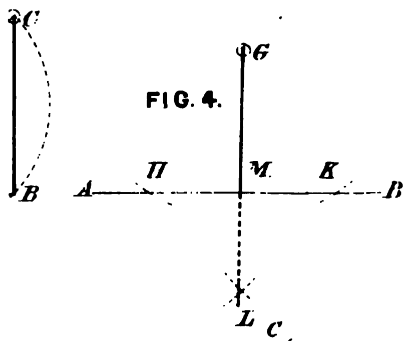


FIG. 4.

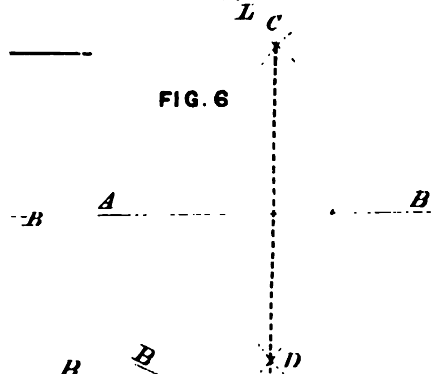


FIG. 6

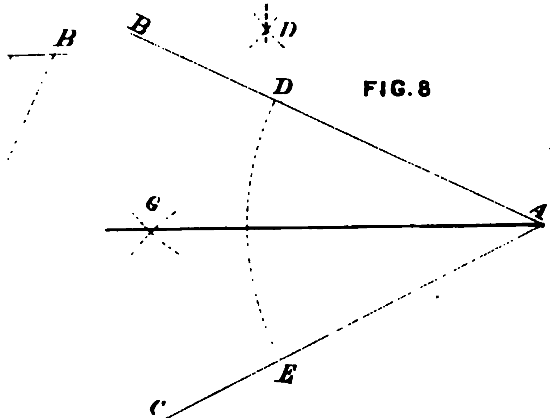
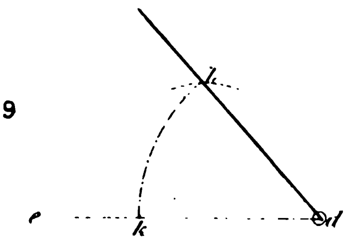


FIG. 8

FIG. 9



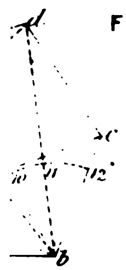


FIG. 10

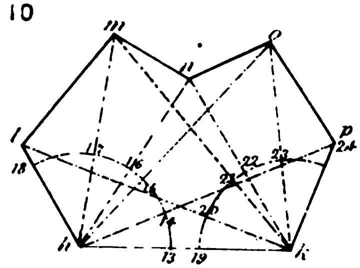


FIG. 13

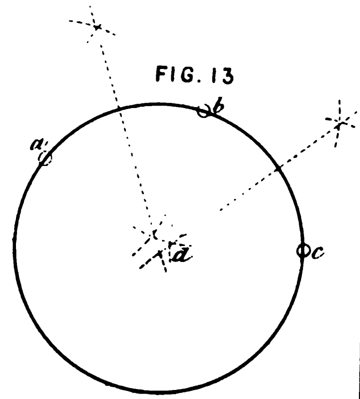
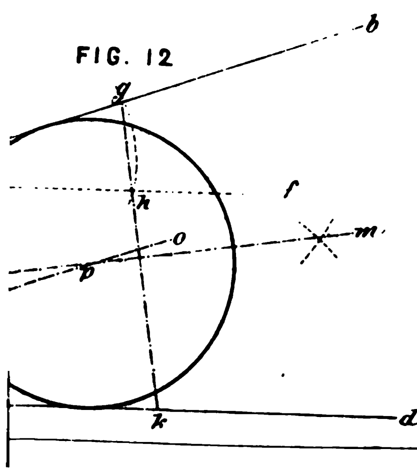


FIG. 12



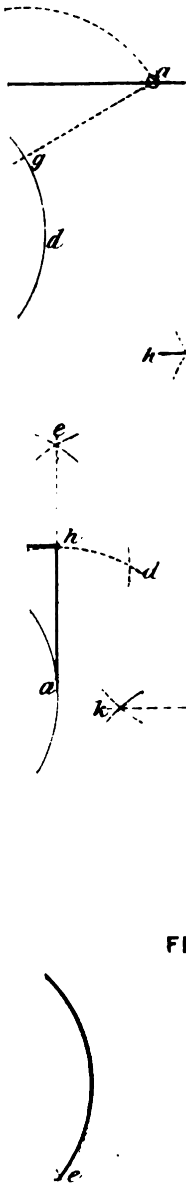


FIG. 15

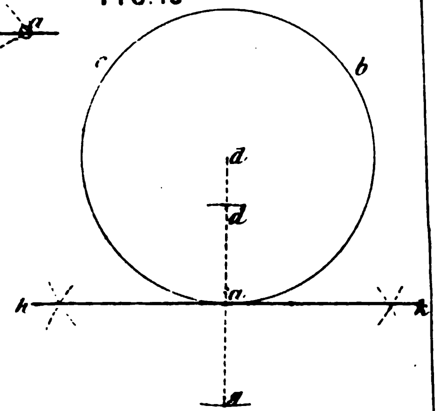


FIG. 17

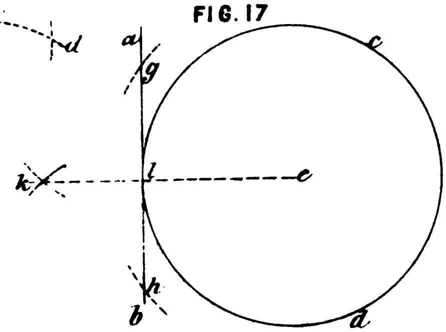


FIG. 19

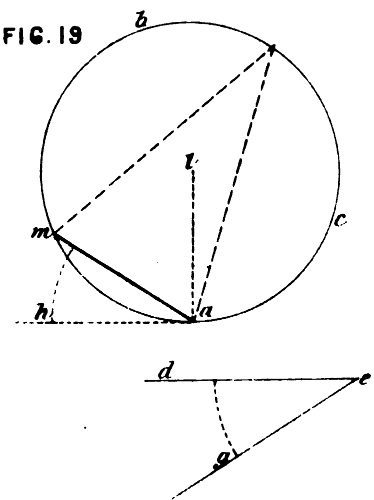


FIG. 21

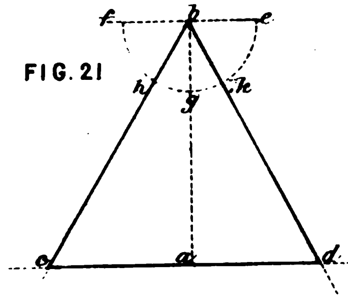


FIG. 23

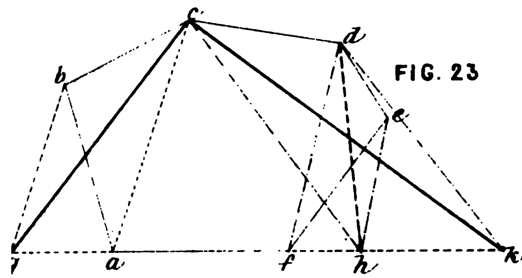
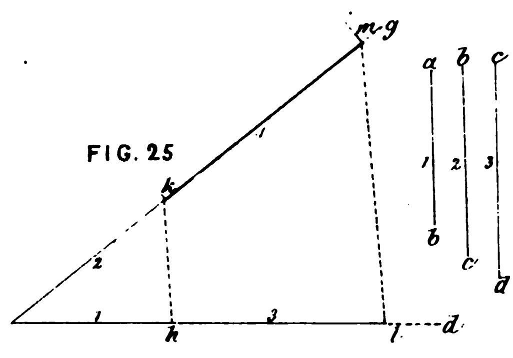
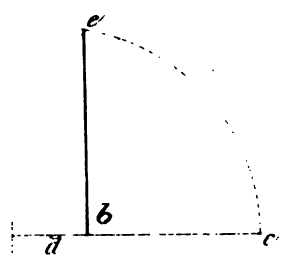


FIG. 25





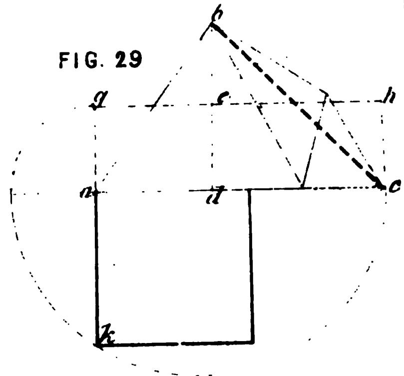


FIG. 29

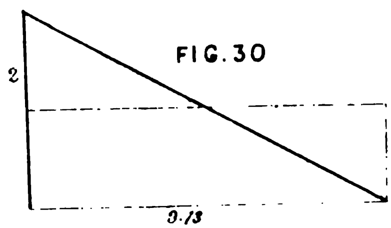


FIG. 30

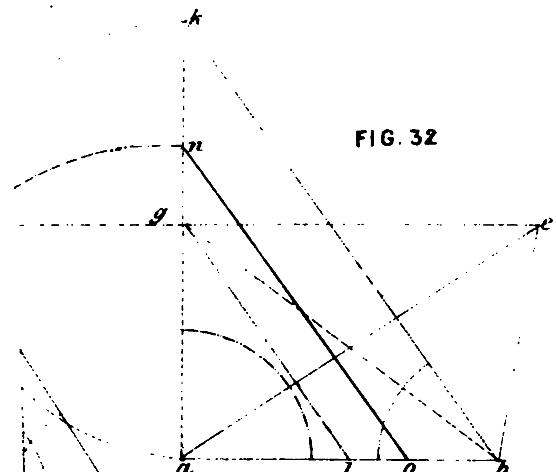
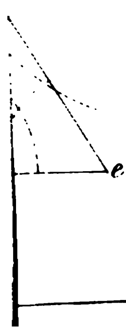
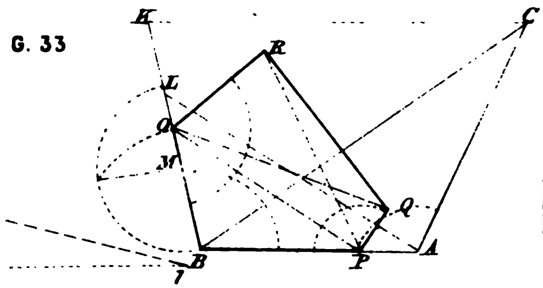


FIG. 32



G. 33



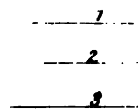


FIG. 35

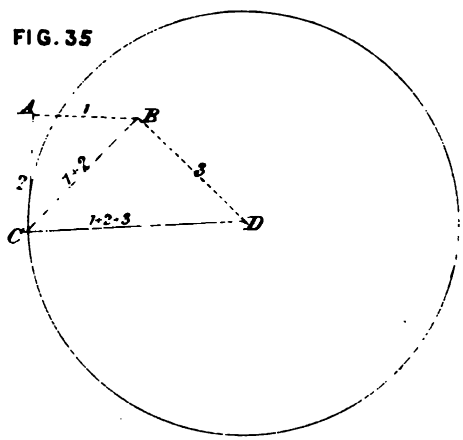


FIG. 36

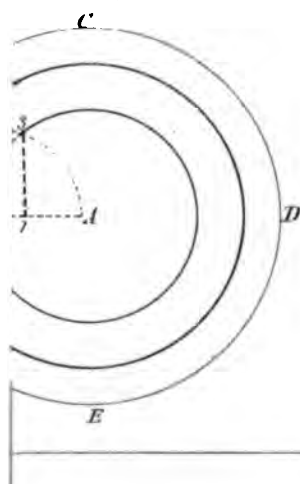


FIG. 38

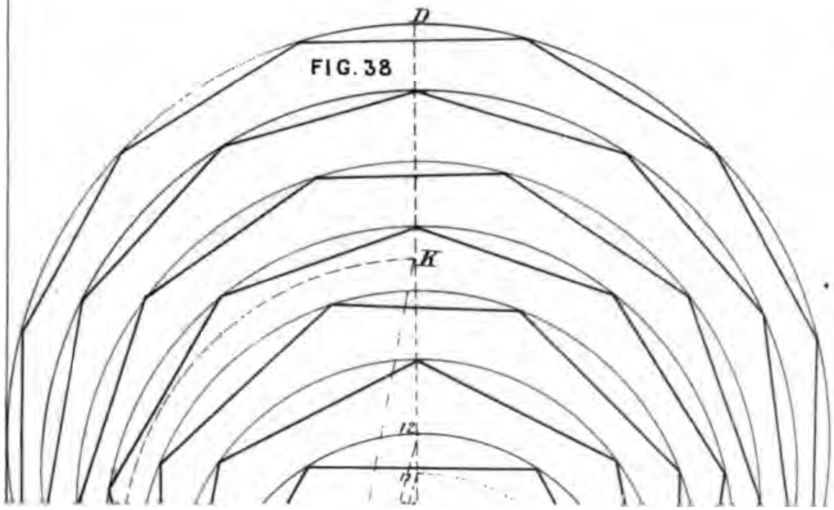


FIG. 37

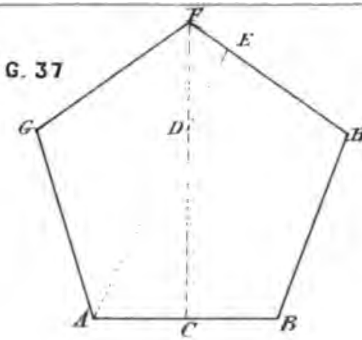


FIG. 39

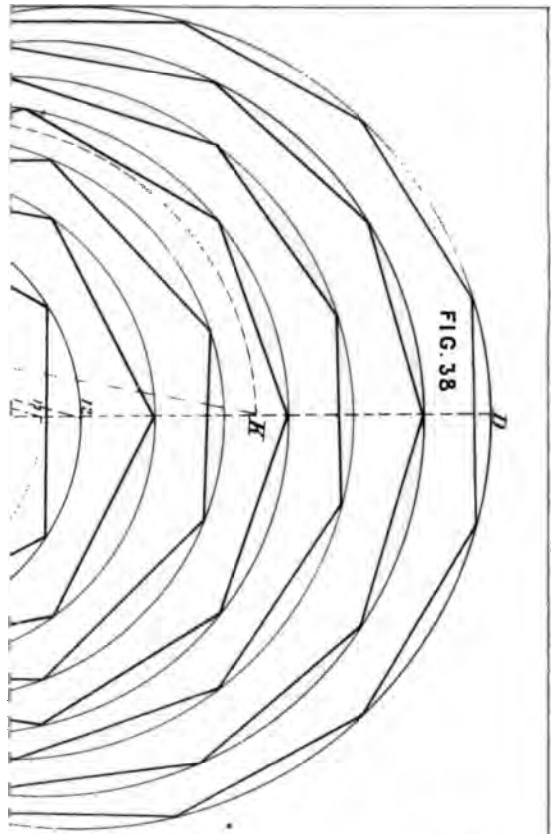


FIG. 38

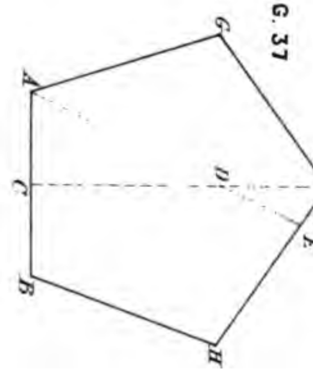


FIG. 37

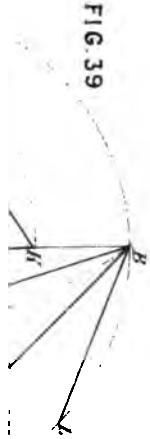
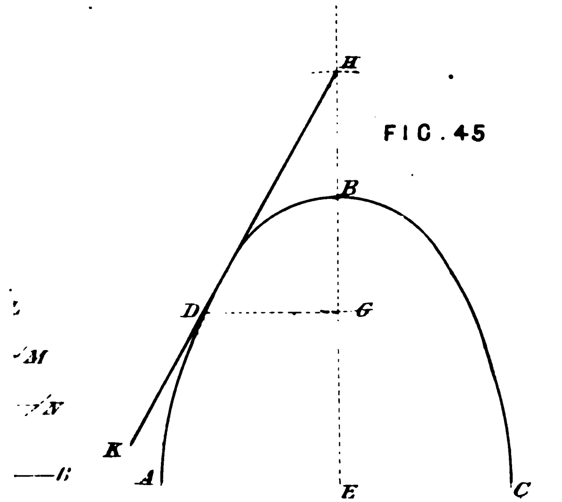
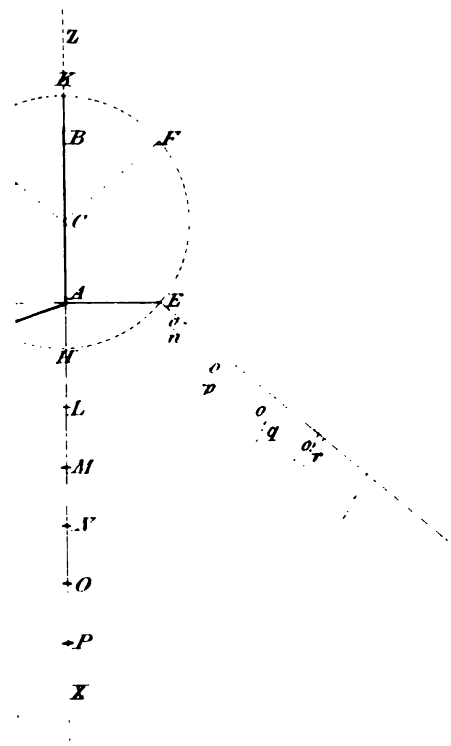


FIG. 39

FIG. 45





imations

FIG. 48

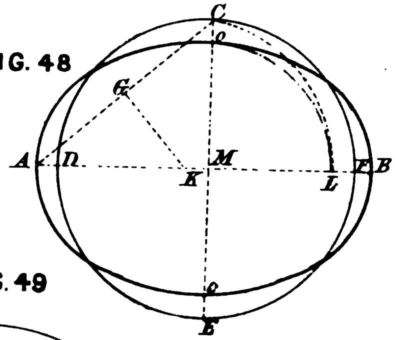


FIG. 49

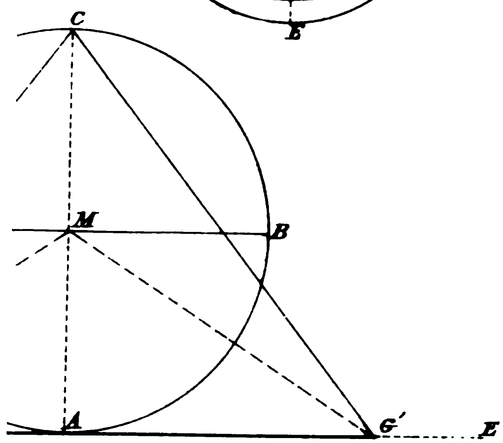


FIG. 50

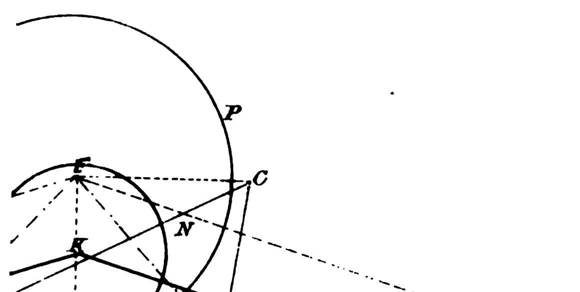


FIG. 1

$\frac{1}{144}$

FIG. 2

$\frac{1}{1440}$

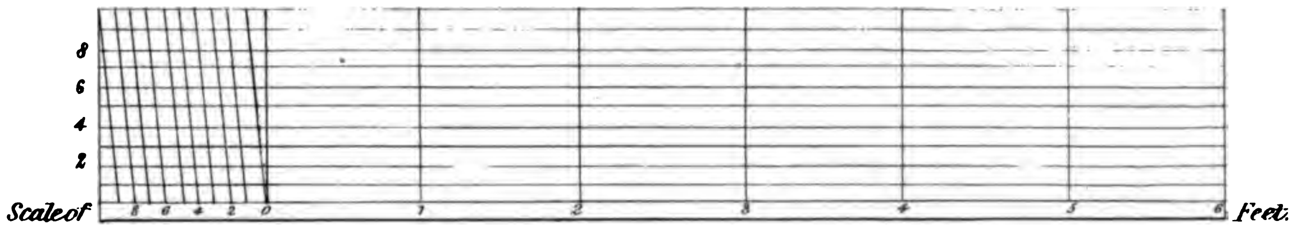


FIG. 3

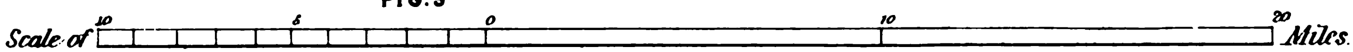
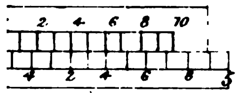


FIG. 3

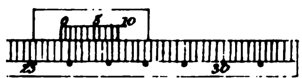
Verniers.

nches and $\frac{1}{100}$ths of Inches.



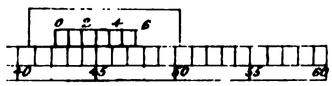
Shows 3.85 Inches.

d Degrees and Minutes.



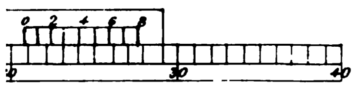
Shows 25°. 50′

ad Fathoms and Feet.



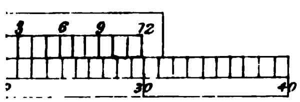
Shows 42 Fathoms, 3 Feet

Miles and Furlongs.



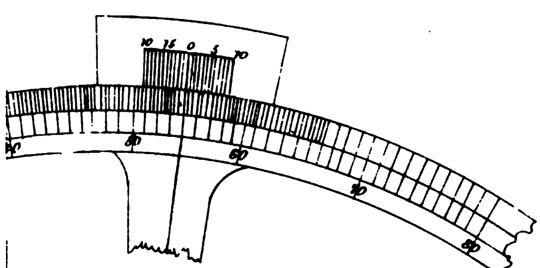
Shows 20 Miles, 5 Furl.

d Feet and Inches.

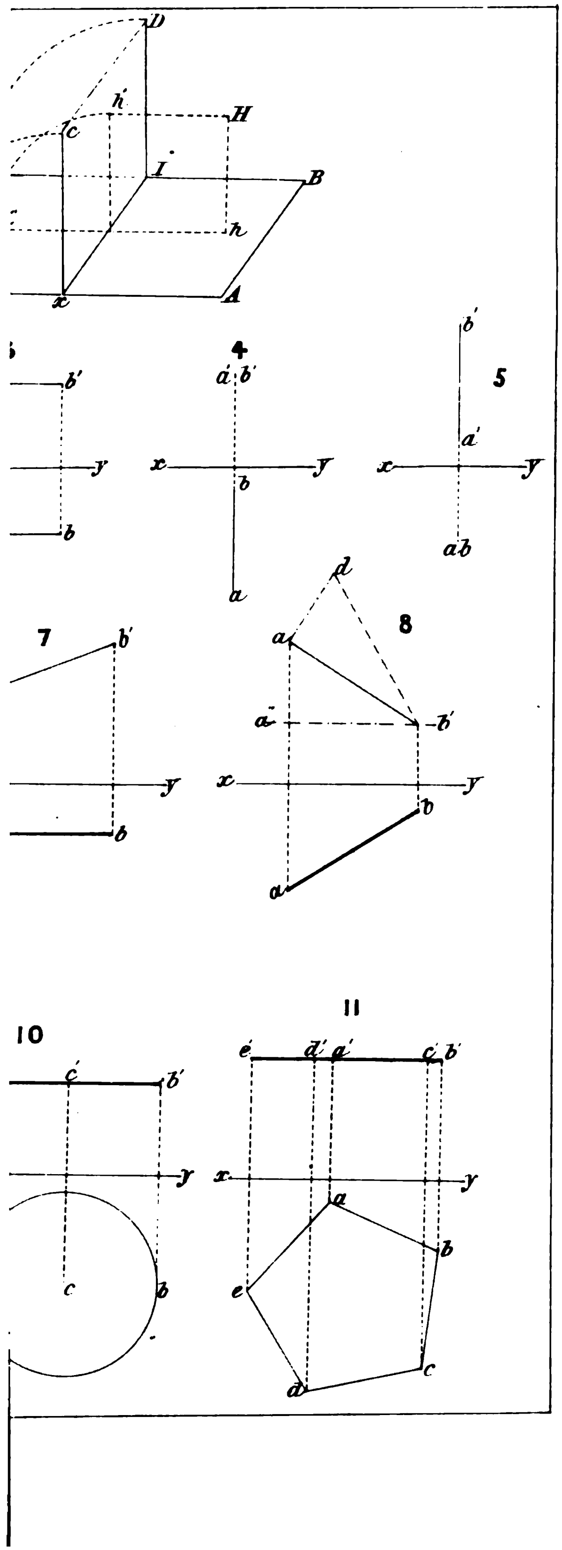


Shows 18 Feet, 4 Inches.

ad Degrees and Minutes.



Shows 54°. 5′


D
h'
H
c
I
B
h
x
A
4
a' b'
b'
5
a'
x
y
b
ab
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a''
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x
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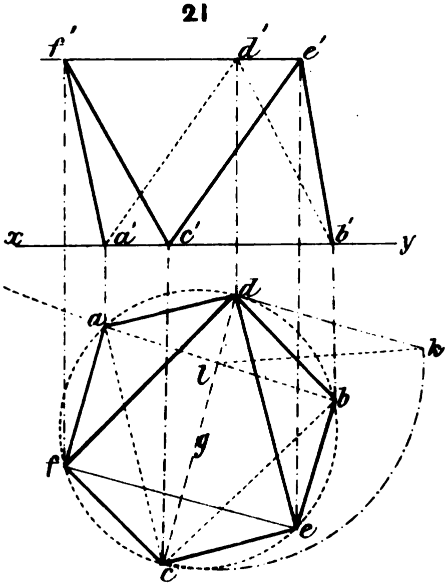
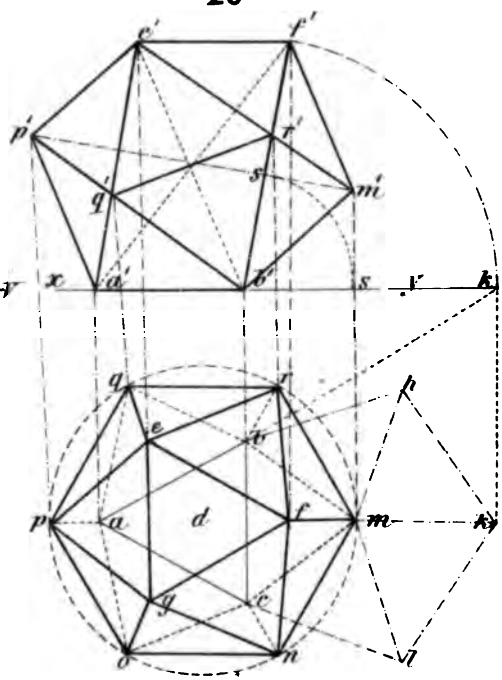
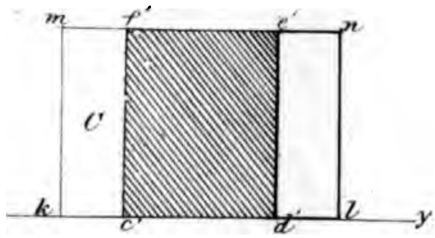
b
a
10
c'
b'
y
c
b
11
e' d' a' c' b'
x
y
a
b
e
c
d

21

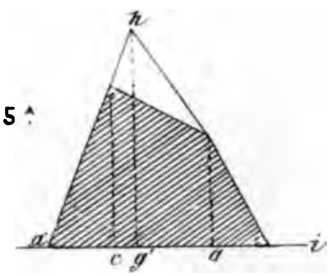


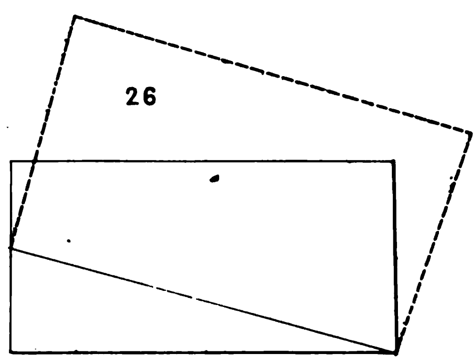
23





25





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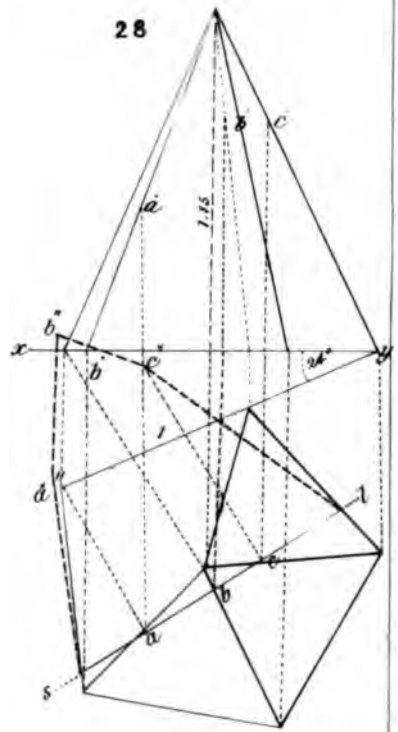

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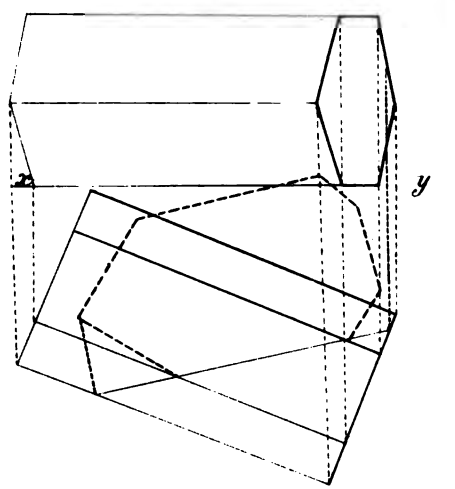

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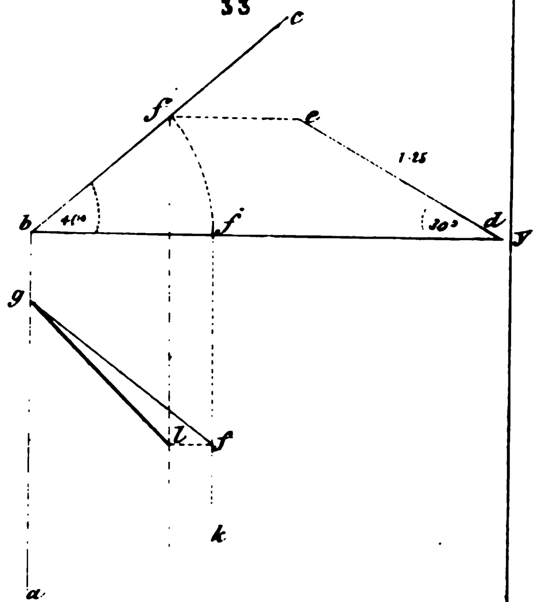

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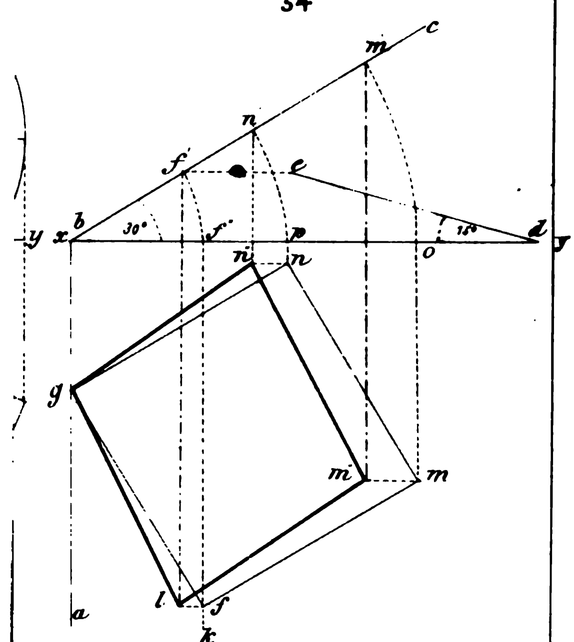
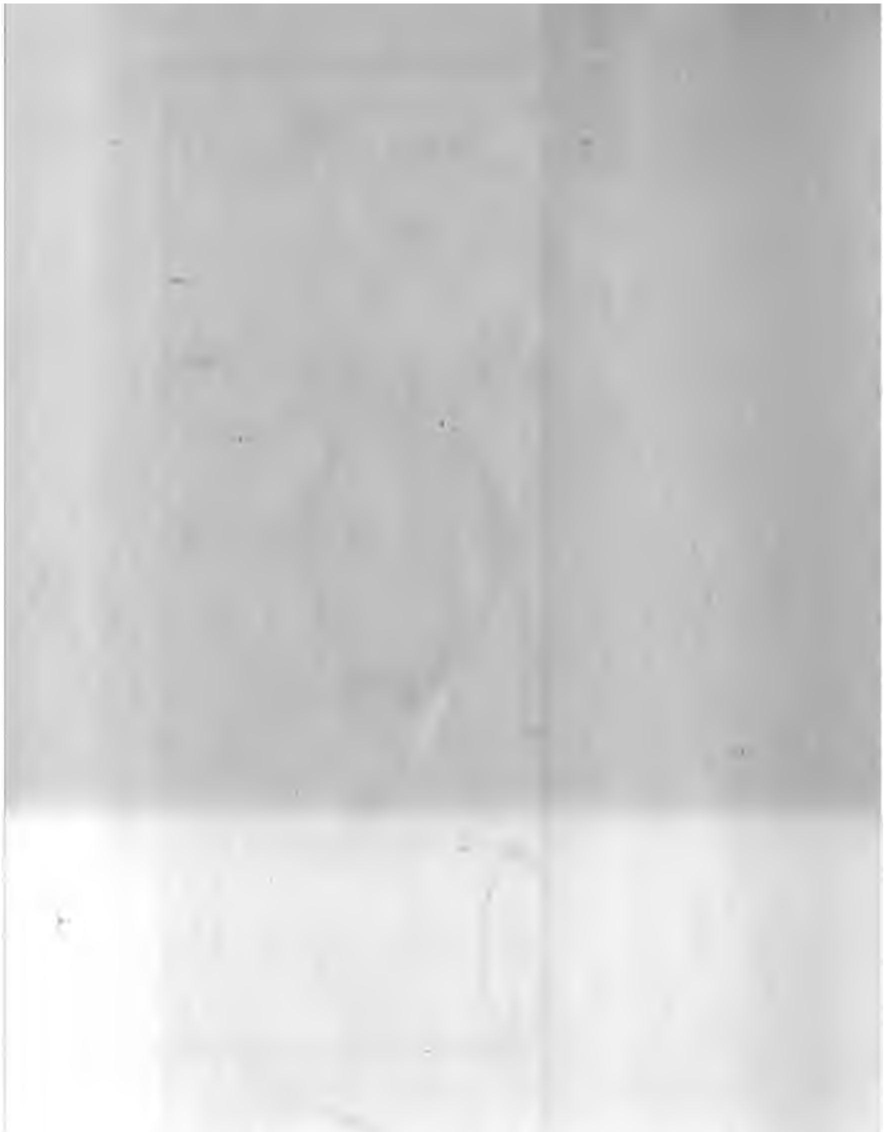
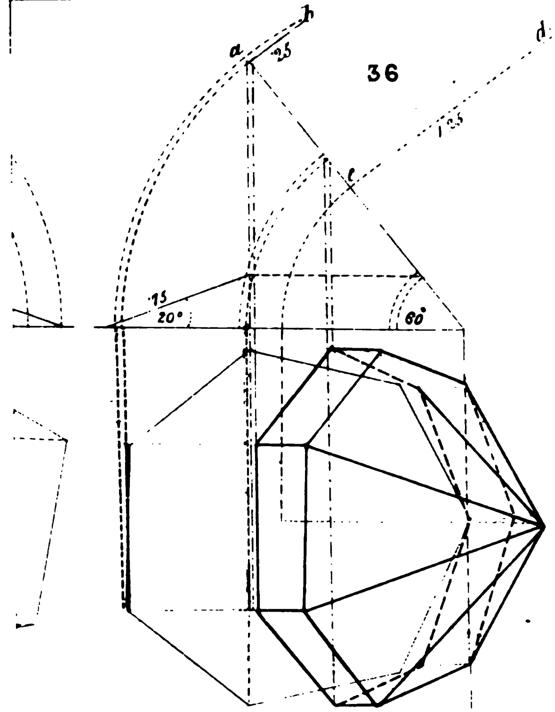




Pl. 98

36



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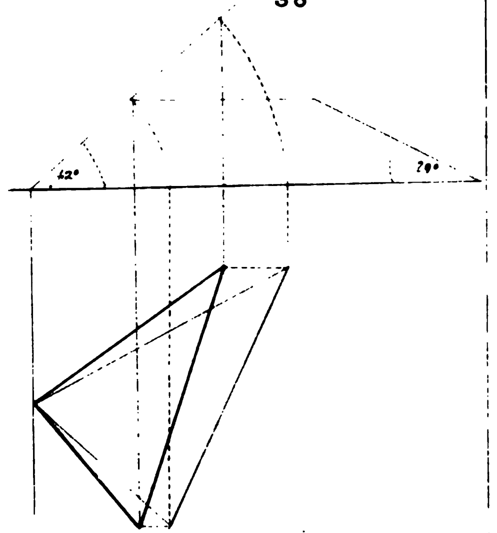
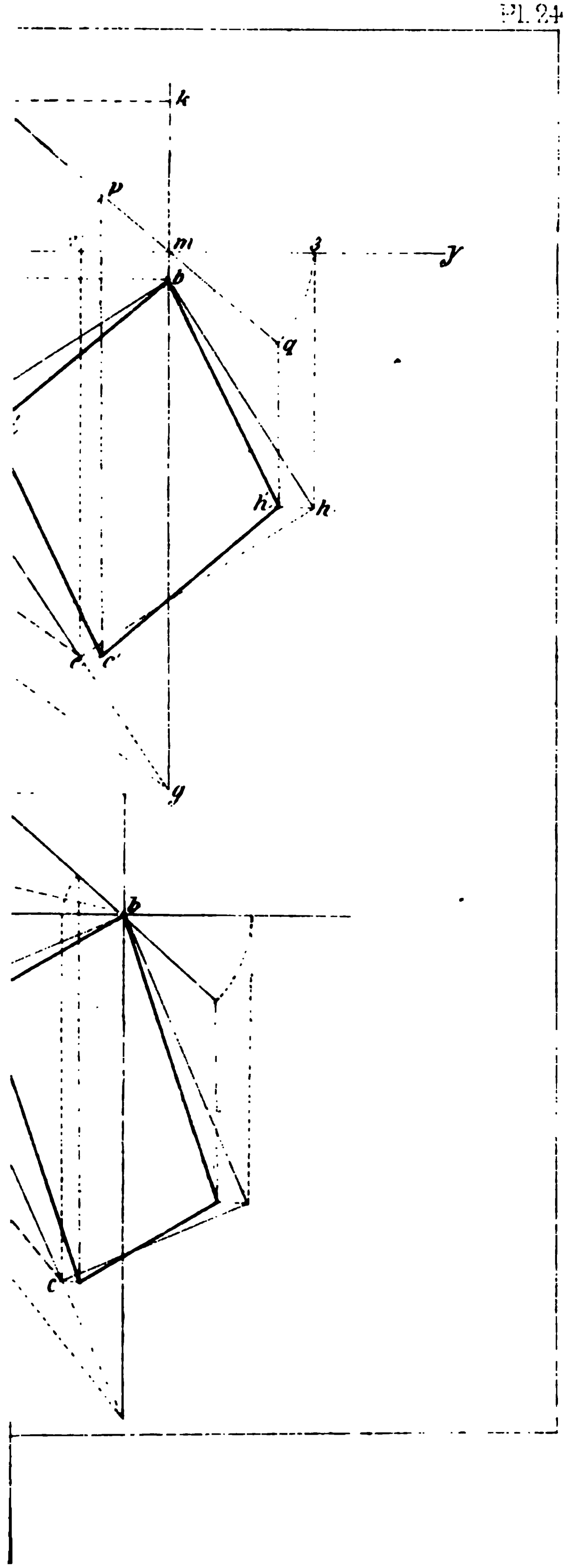
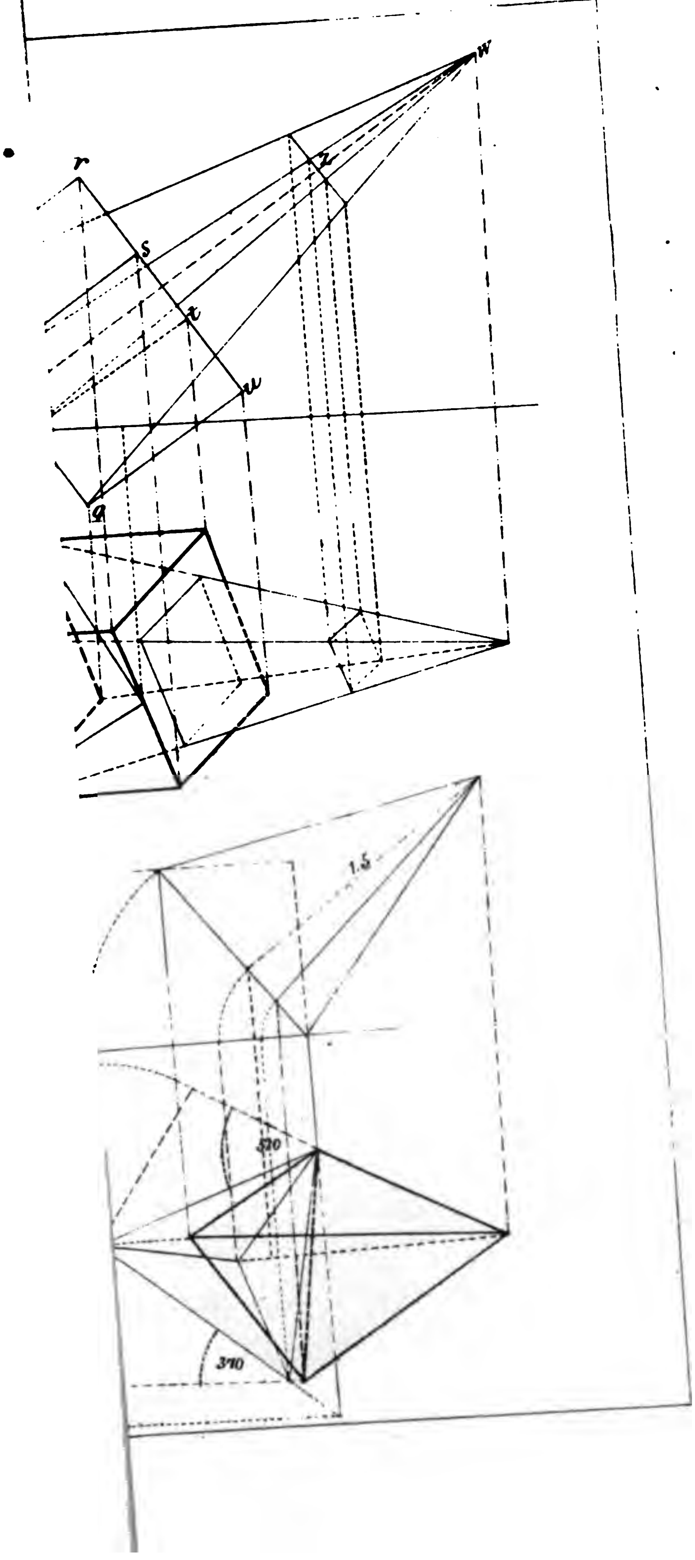
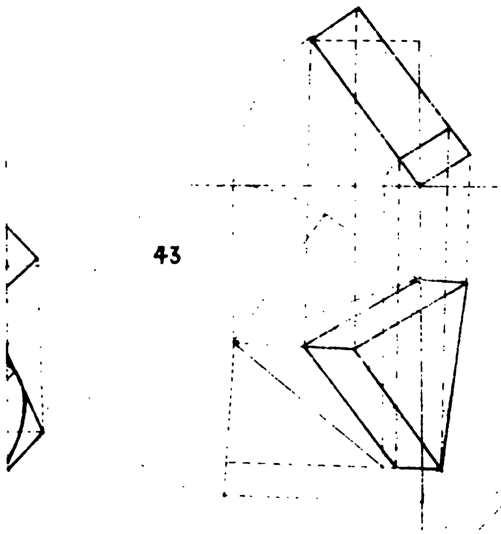



k
p
m
z
y
b
q
h
h
c
c
g
b
c


r
s
t
u
w
z
q
1.5
520
340

Pl. 26



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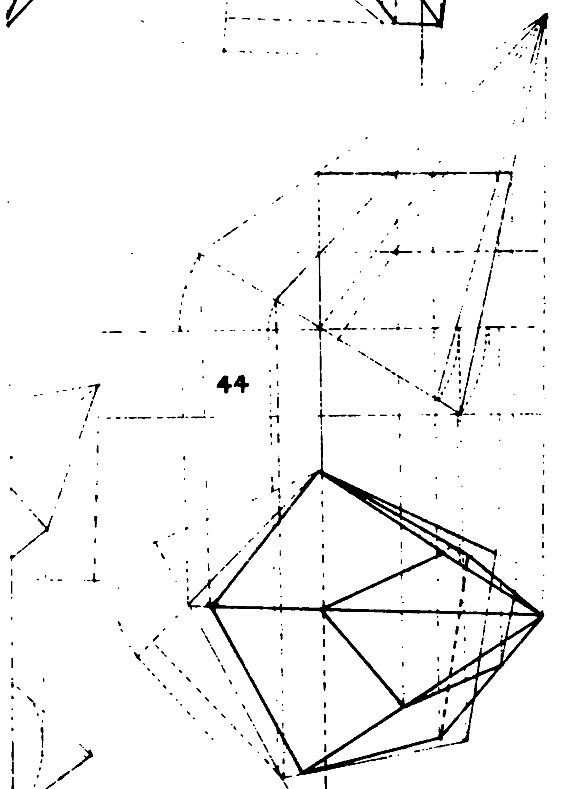

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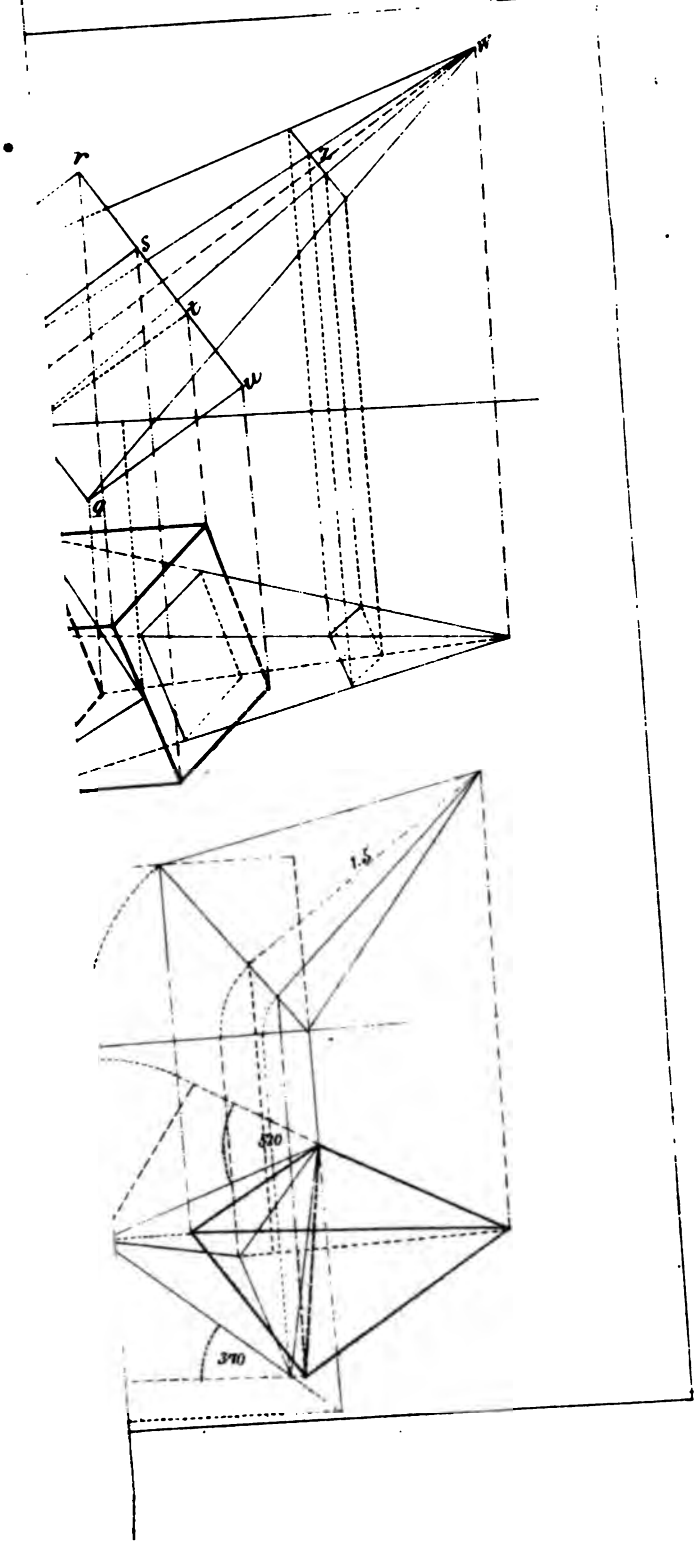
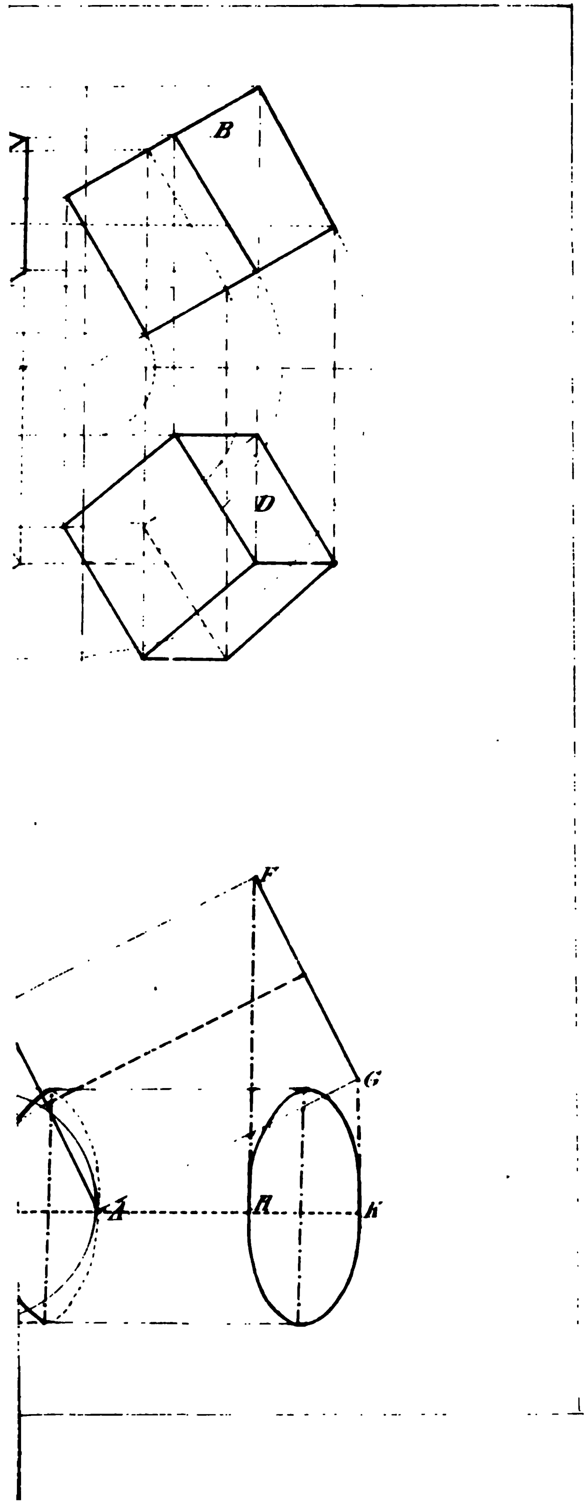
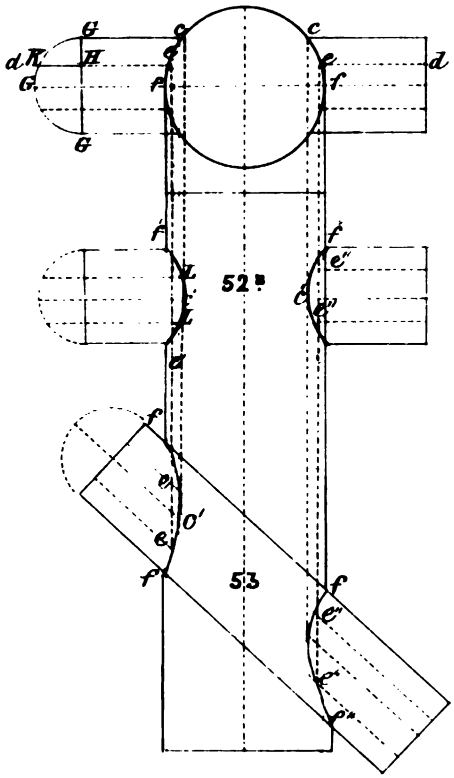

w
r
z
s
t
u
q
1.5
520
370

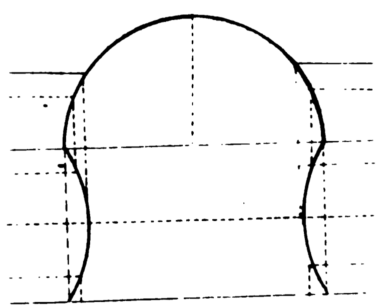


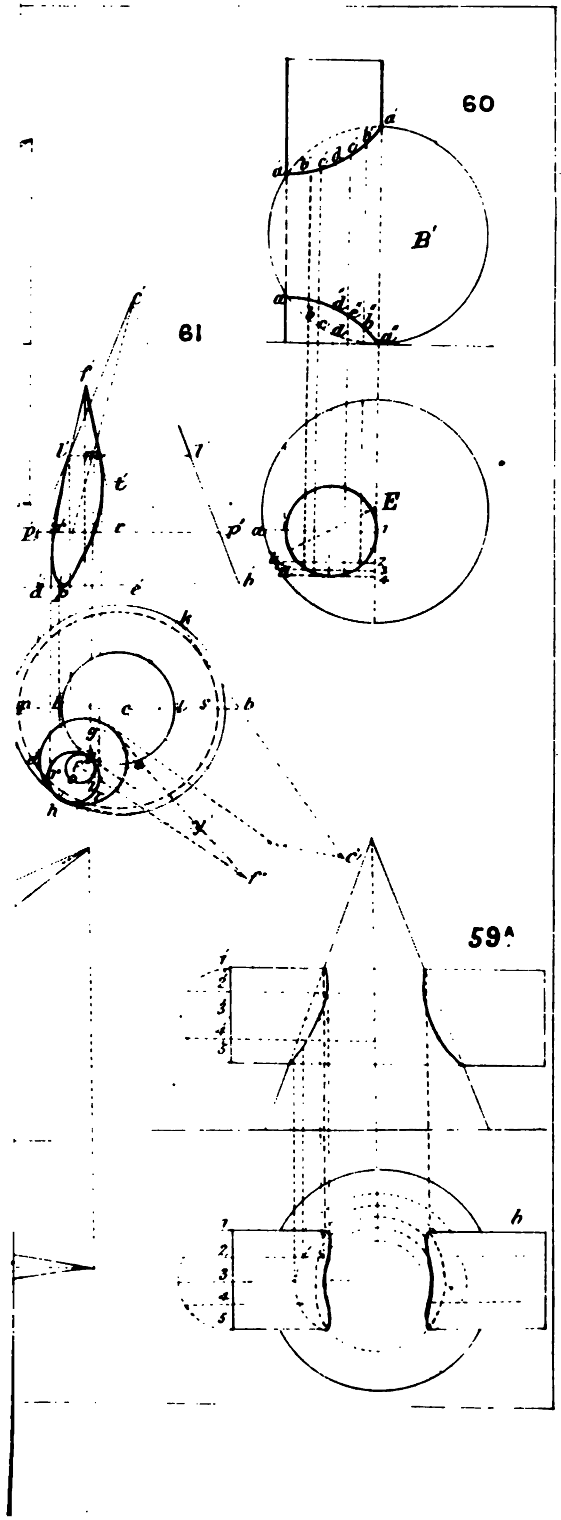
B
D
F
G
A
H
K

52A

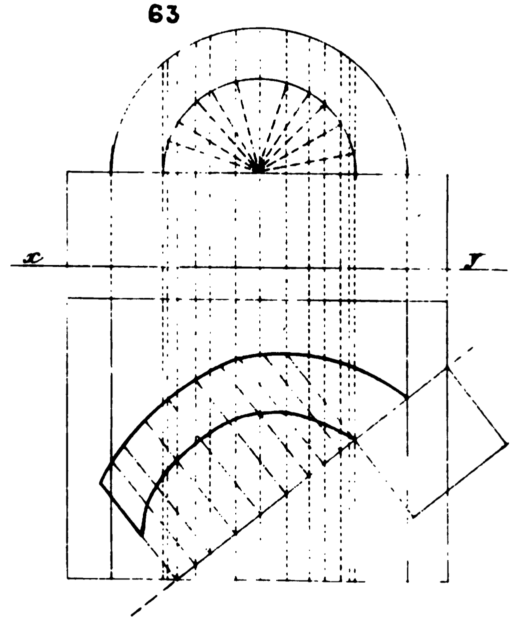


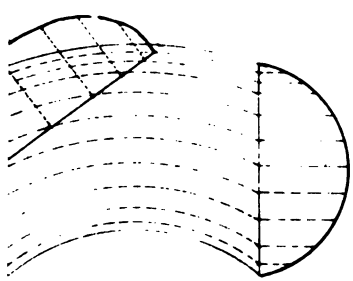
56

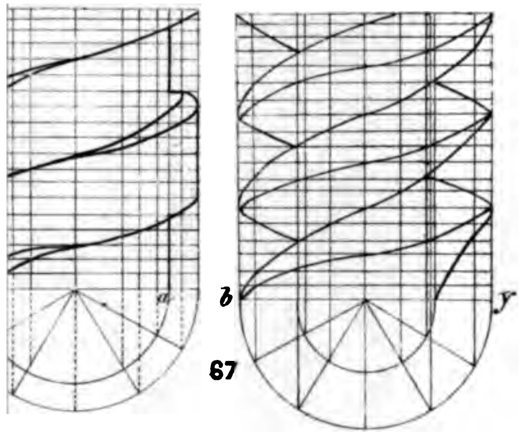



60
B'
E
61
59.A

63

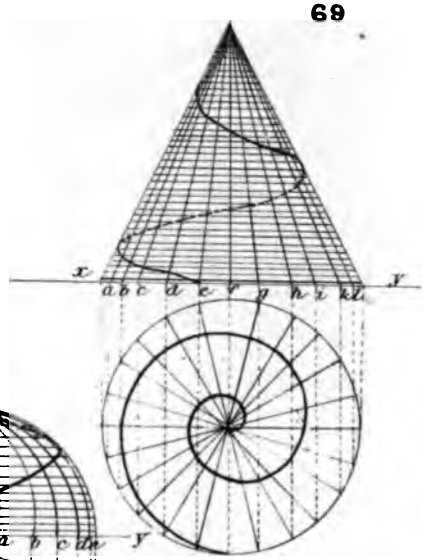






67

69



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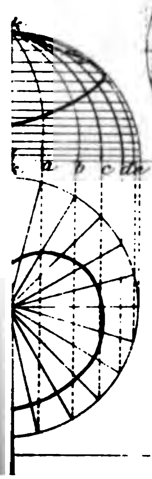
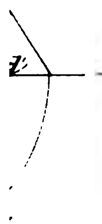
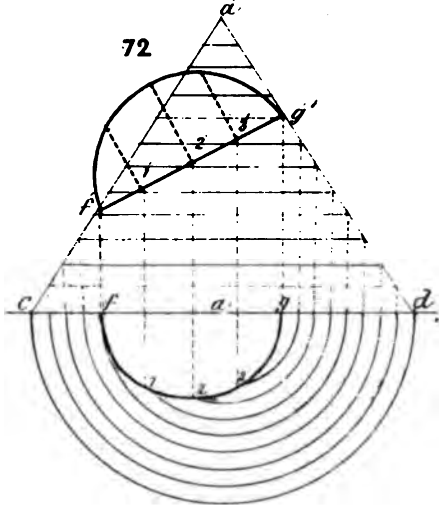
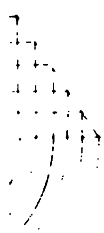


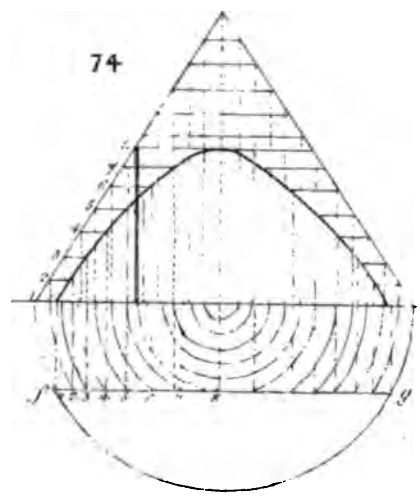


72

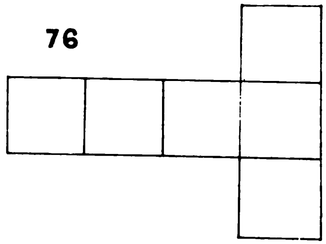




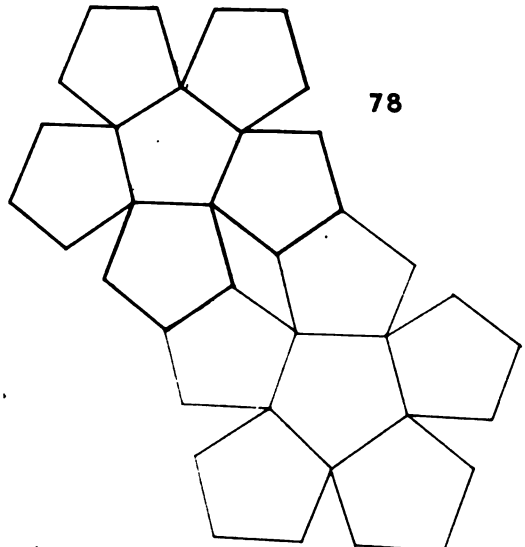
74

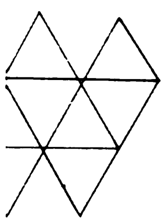


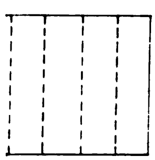
76



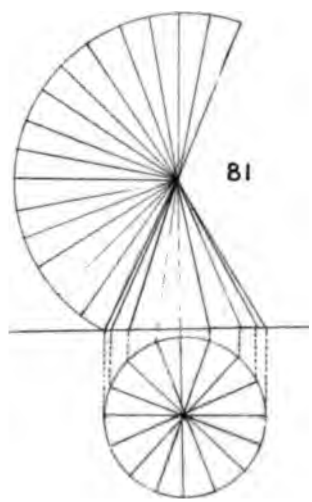
78

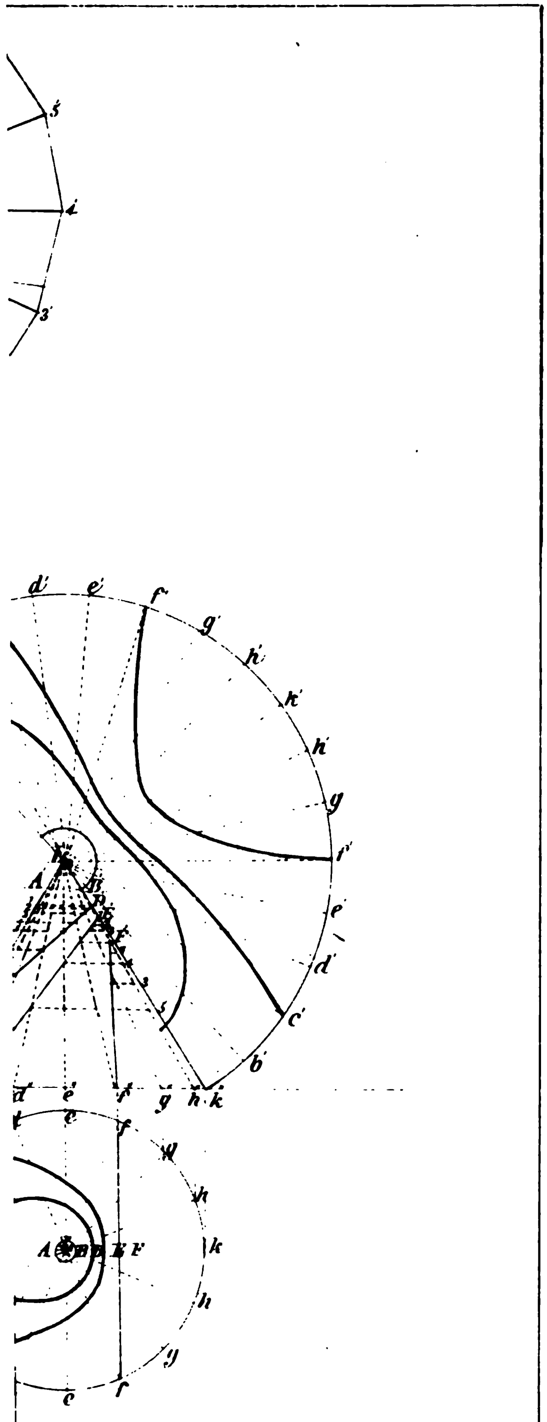






81




5'
4'
3'
d'
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h'
k'
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d'
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f'
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k
e
f
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h
k
h
g
f
e
A
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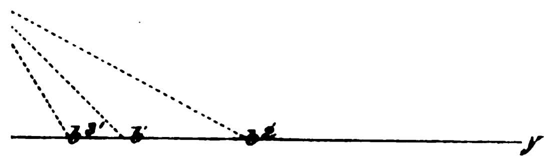
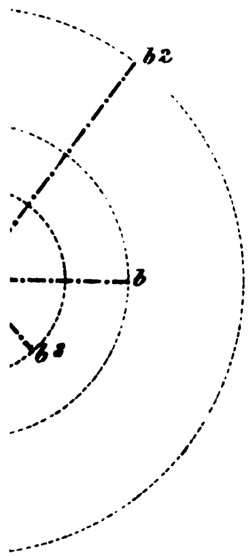
F

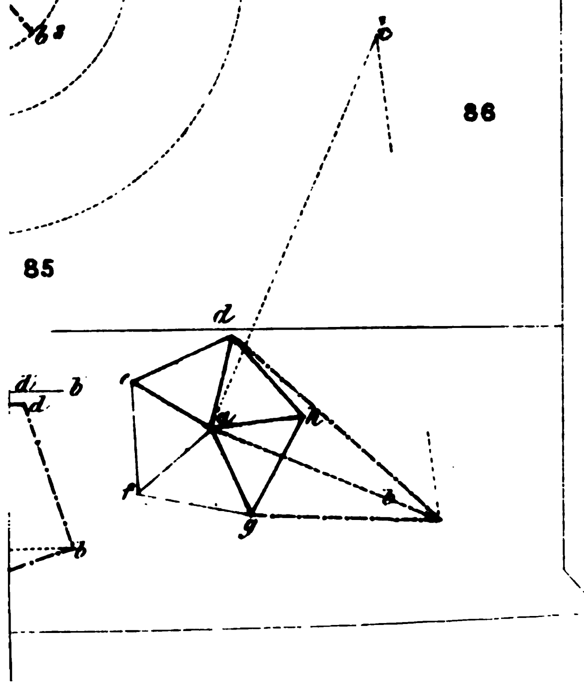
FIG. 84

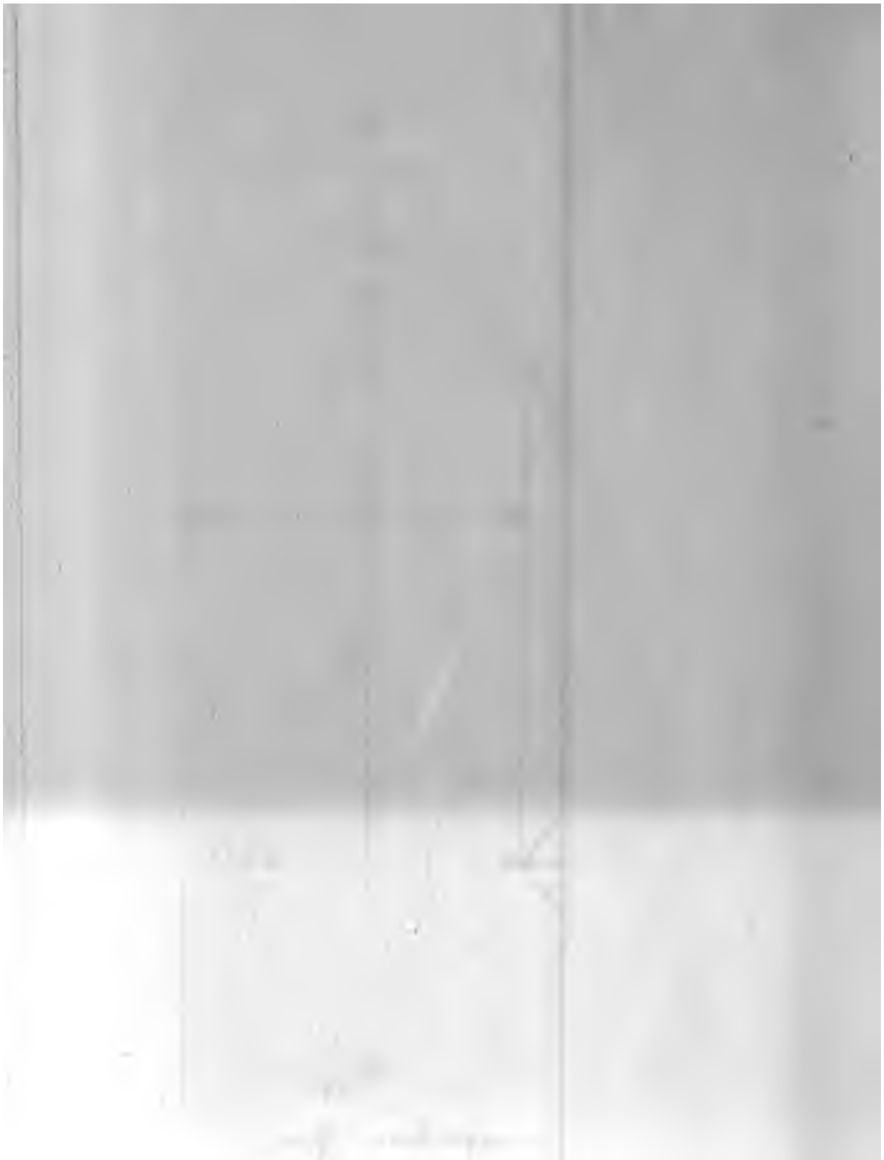




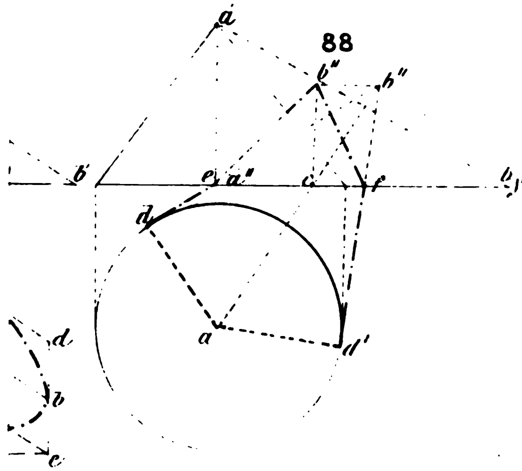
85

86

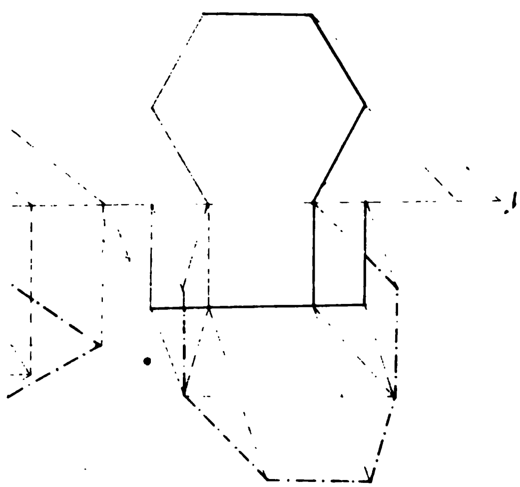




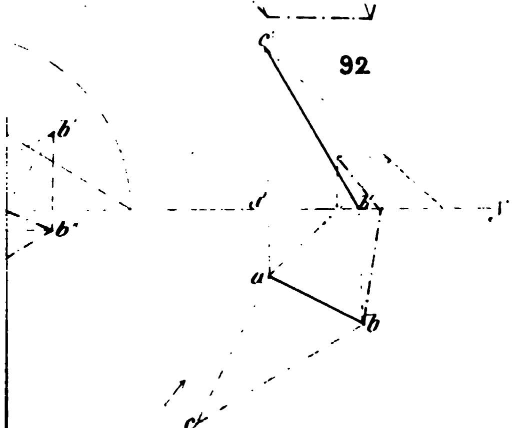
88



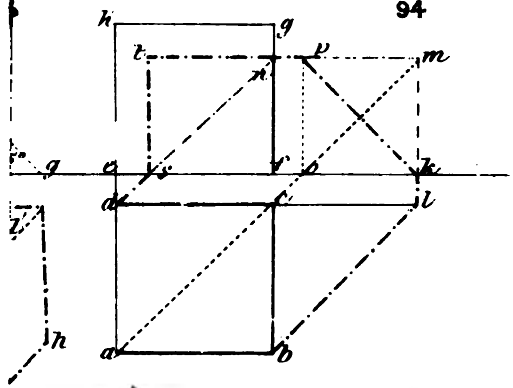
90



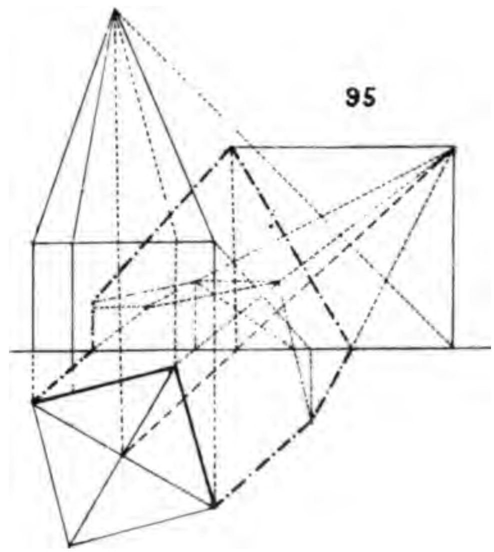
92



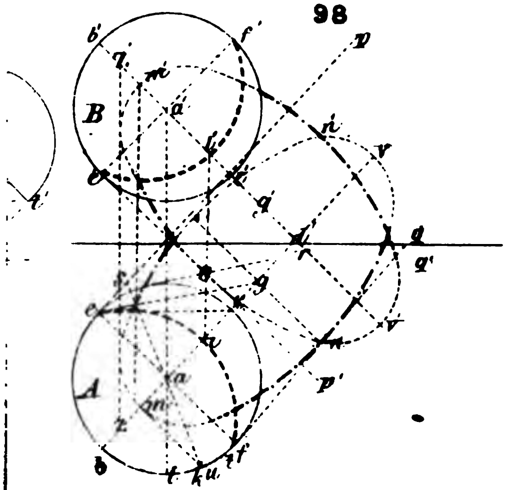
94

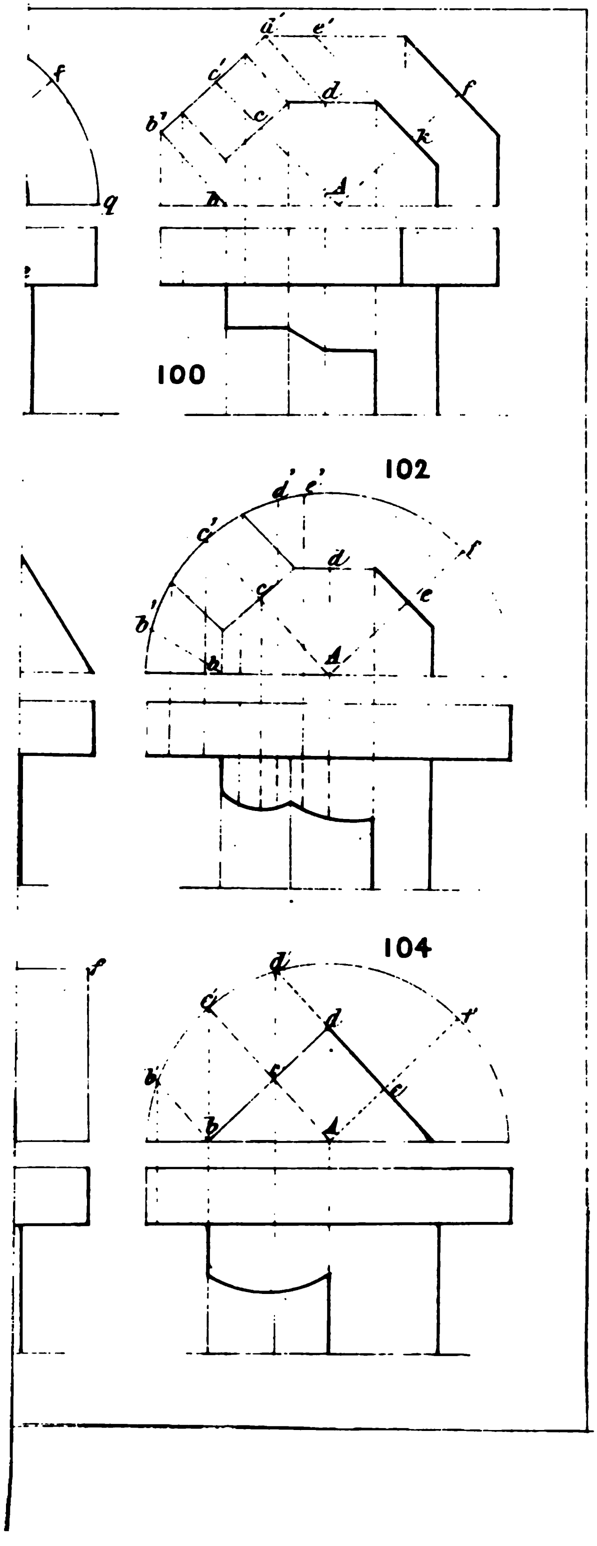


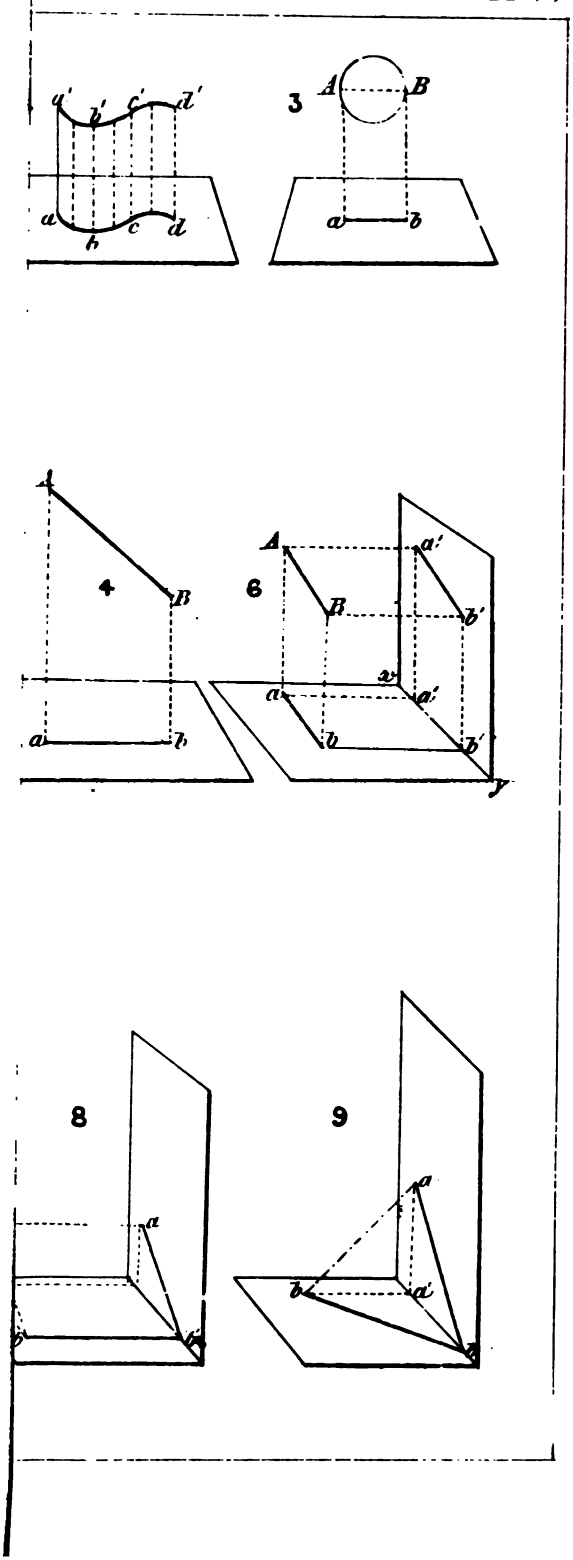
95



98




100
102
104



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A
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c
d
a'
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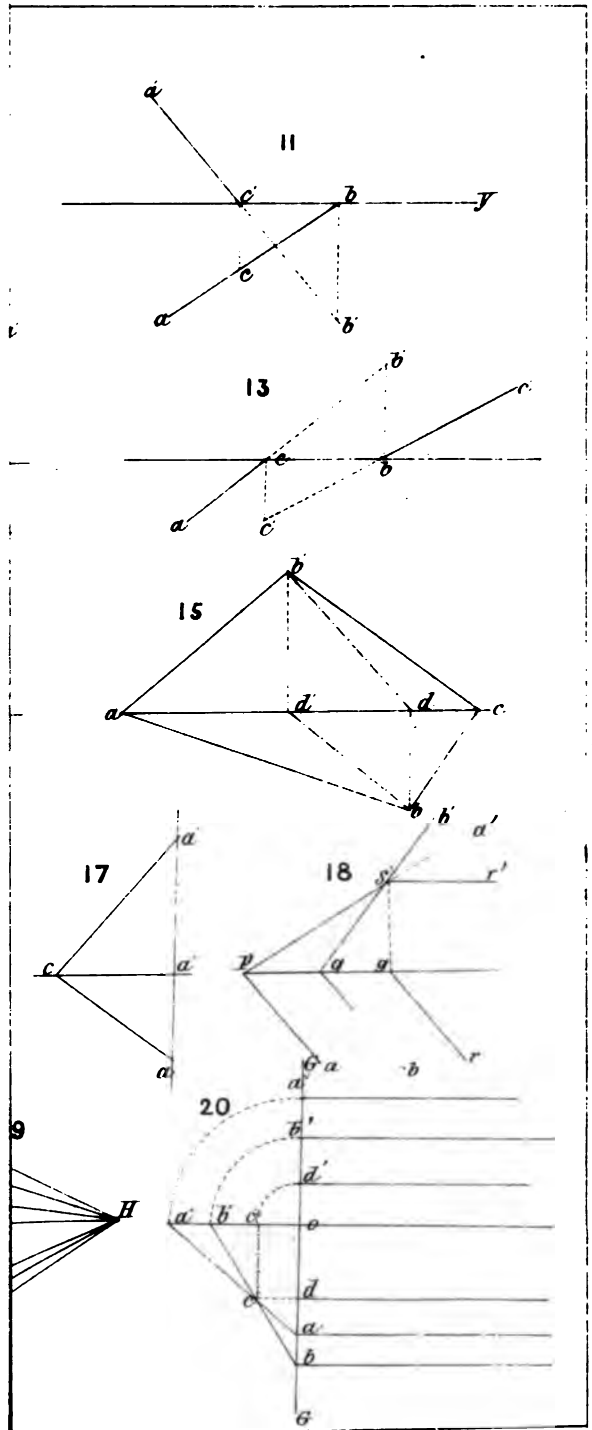
x
y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17
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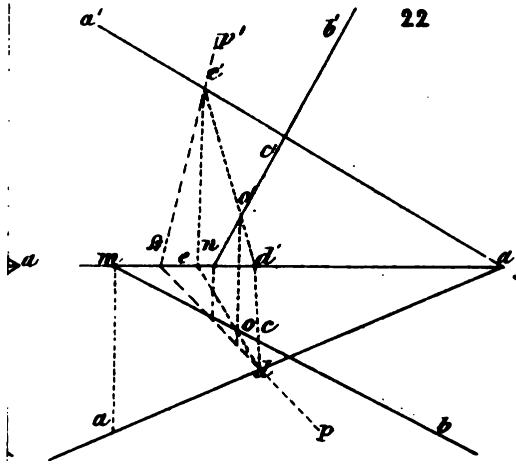
20

22



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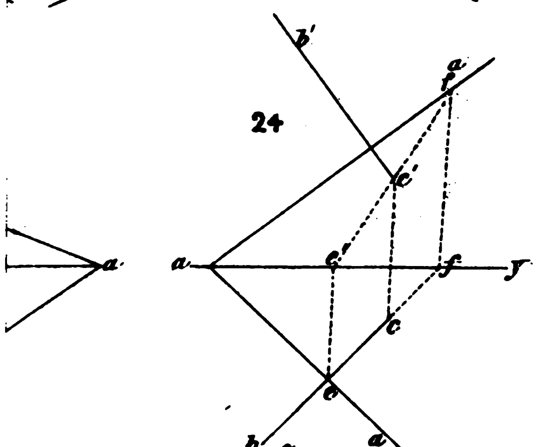

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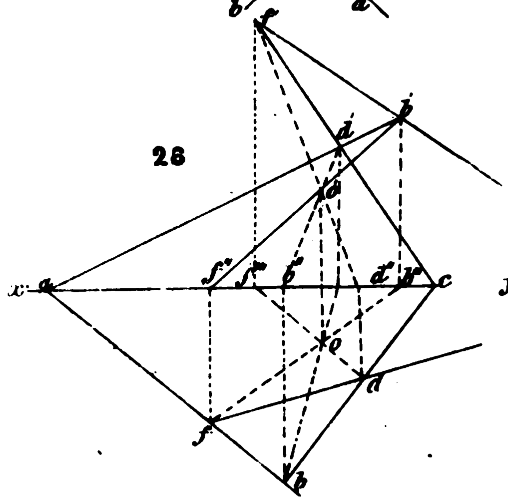

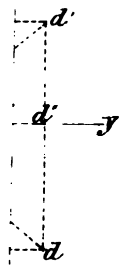

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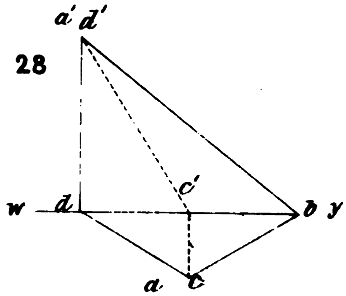

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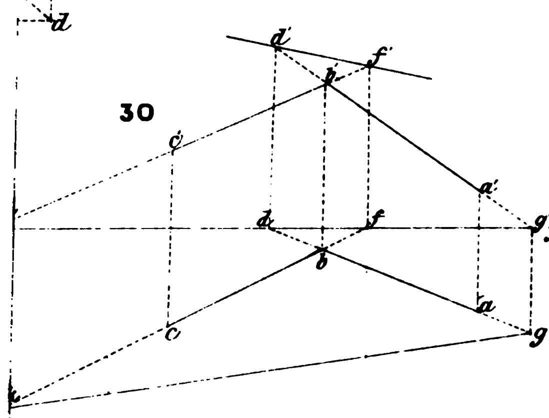

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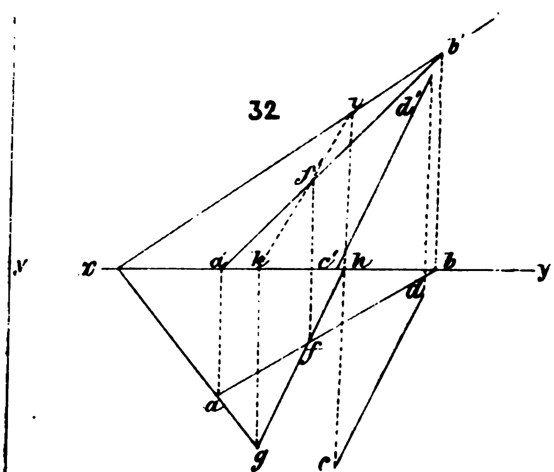

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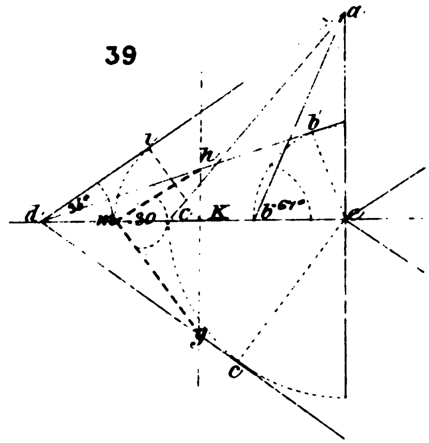

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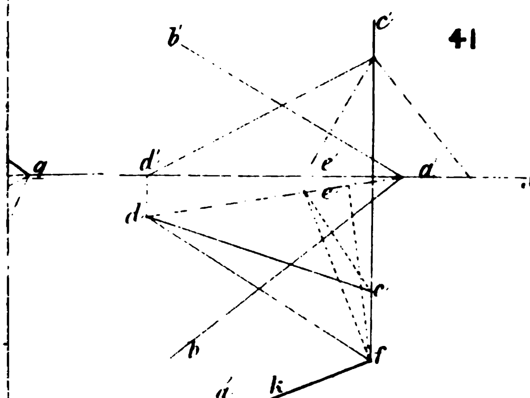

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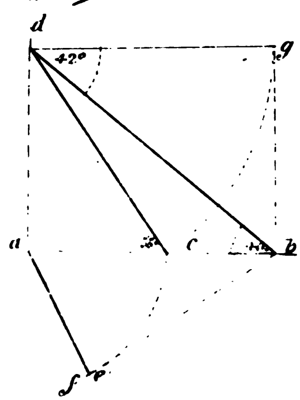
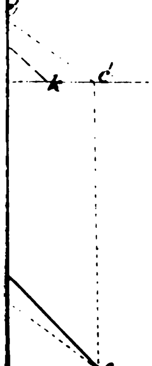
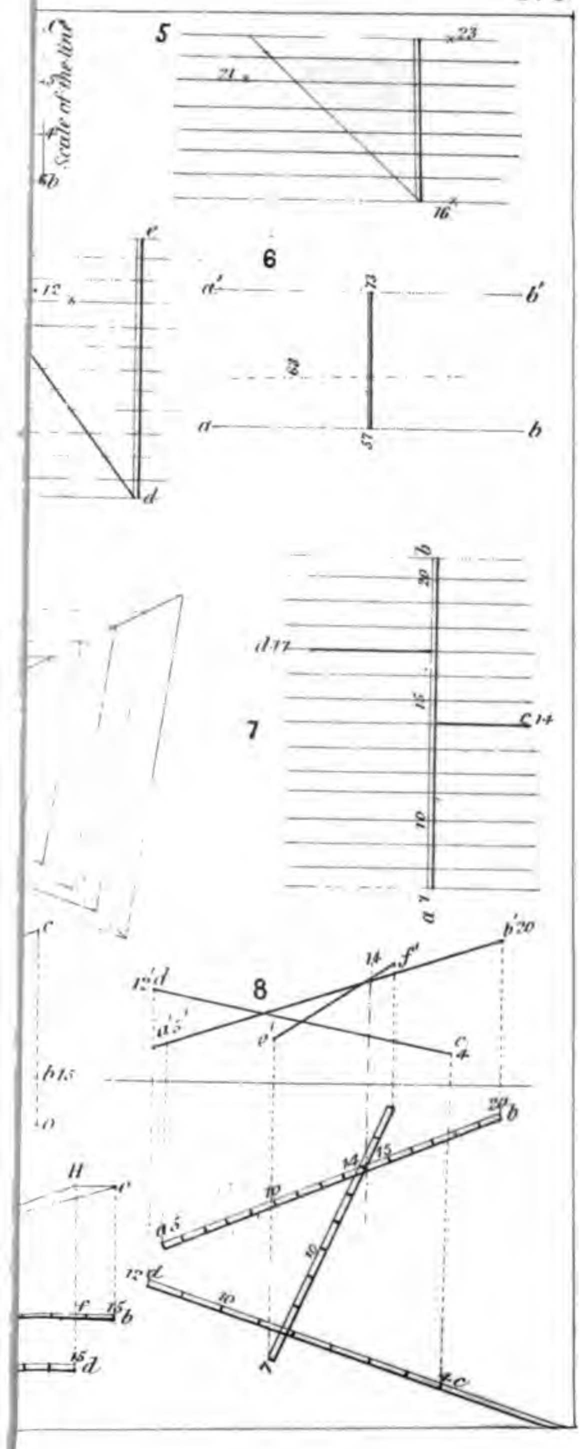
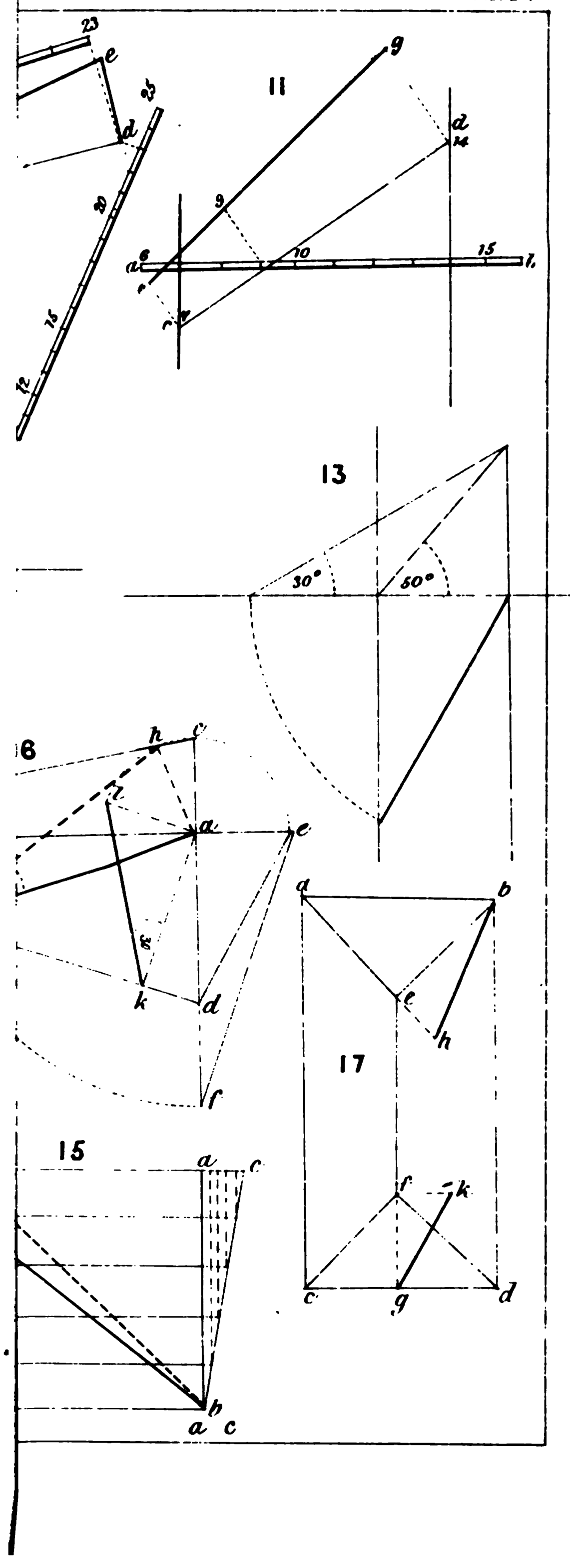





5
6
7
8
Scale of the line
a′
b′
a
b
d
e
c 14
d 17
b′20
f′
e′
c
H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13
30°
50°
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a
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c
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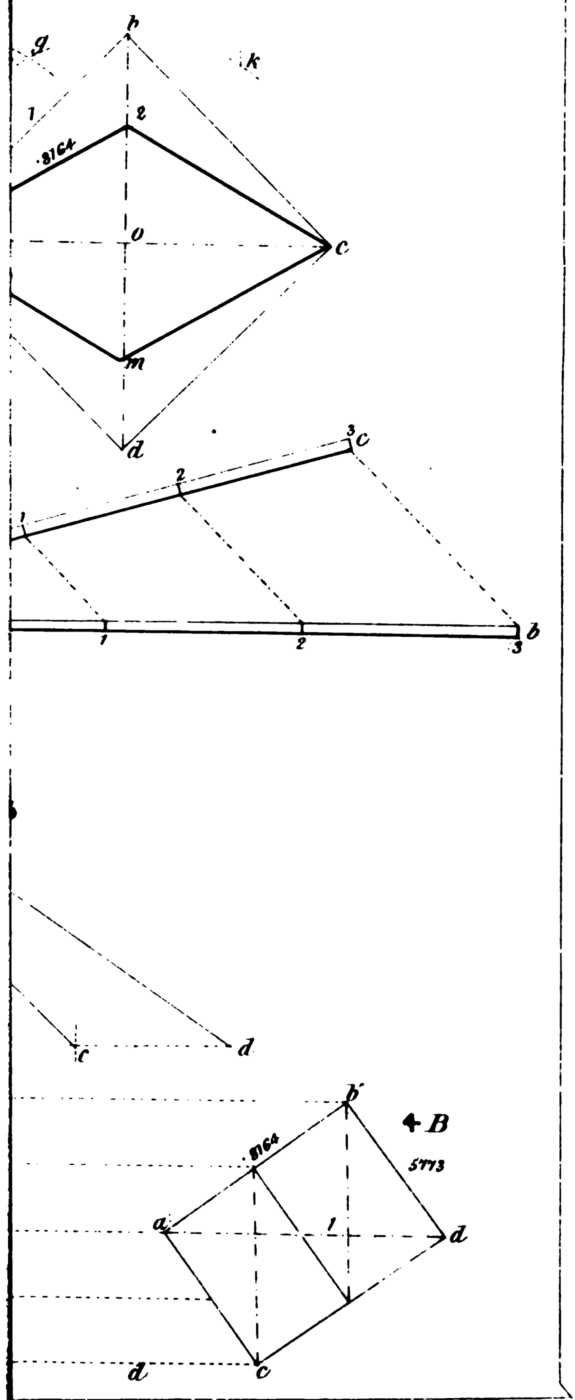
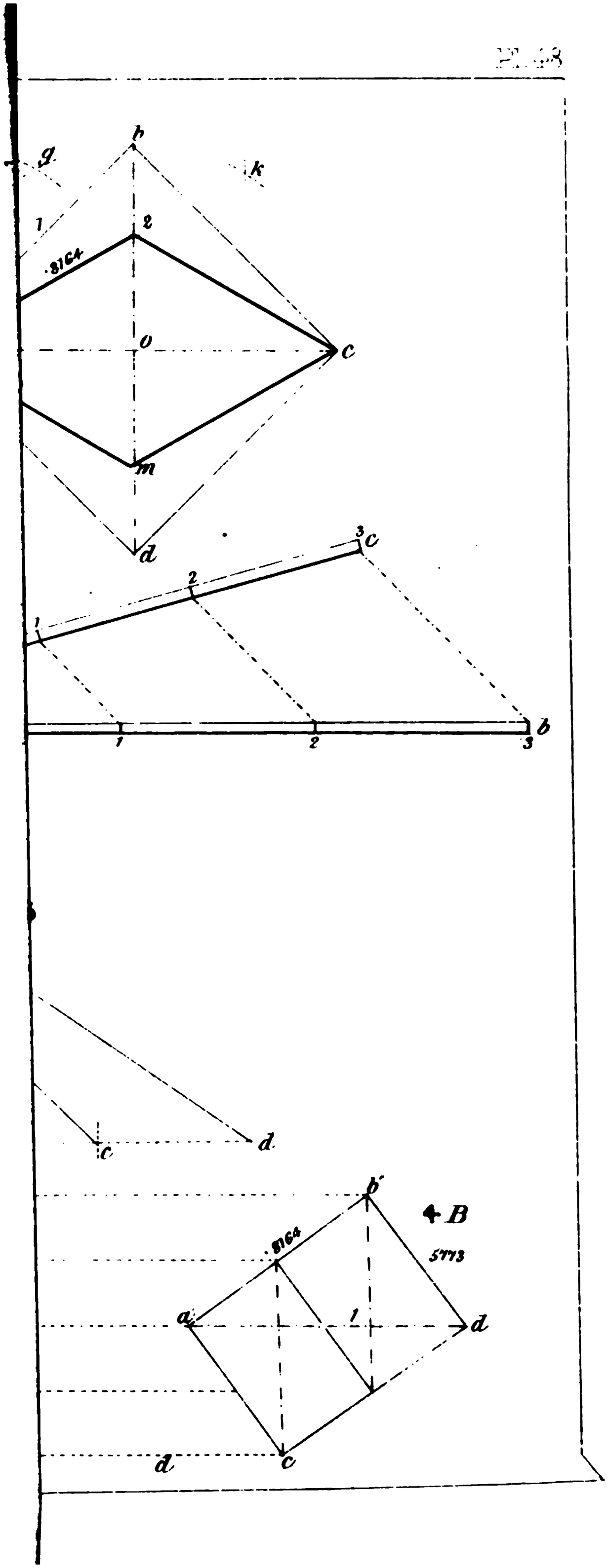
g
h
k
q


g
h
k
1
2
.8164
o
c
m
d
3
c
2
1
1
2
b
3
c
d
b'
+B
.8164
5773
a
1
d
d
c



g
h
k
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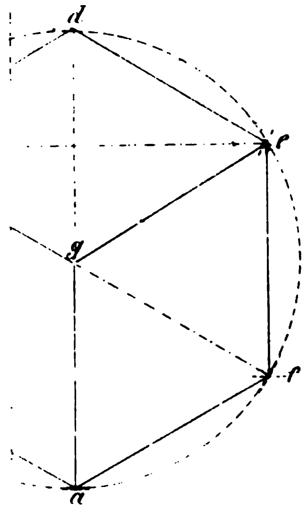
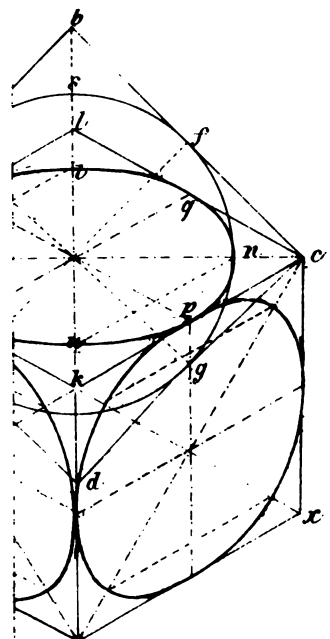
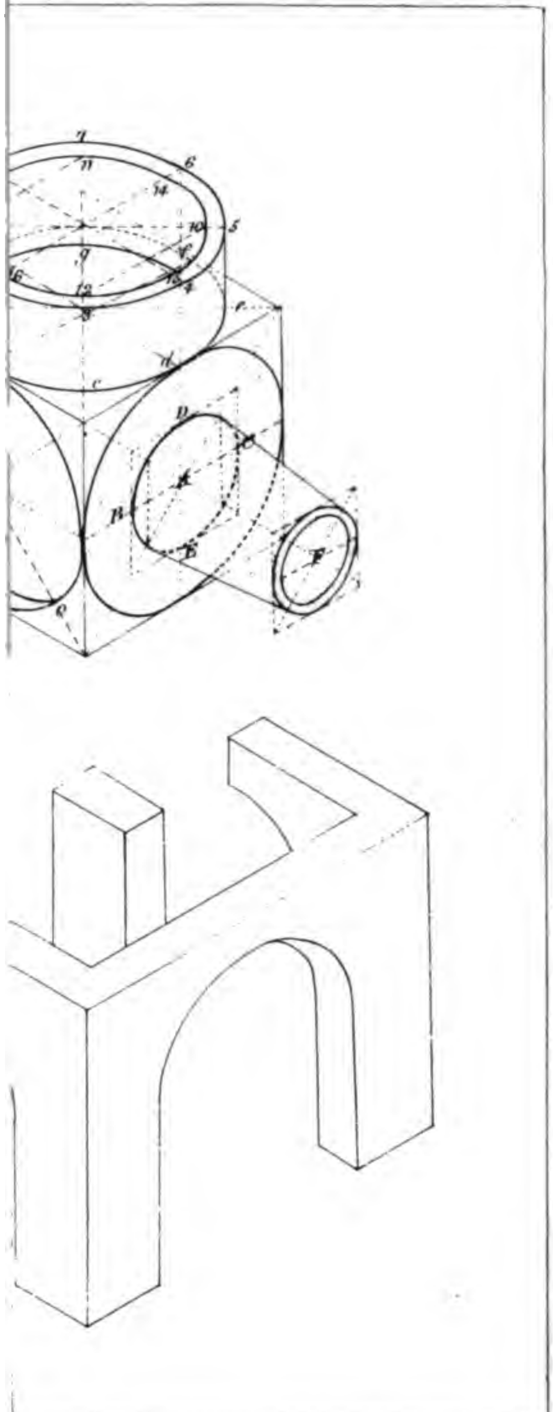
2
.8164
o
c
m
d
3 c
2
1
b
1
2
3
c
d
b'
4 B
.8164
5773
a
1
d
d
c

Pl 50






7
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14
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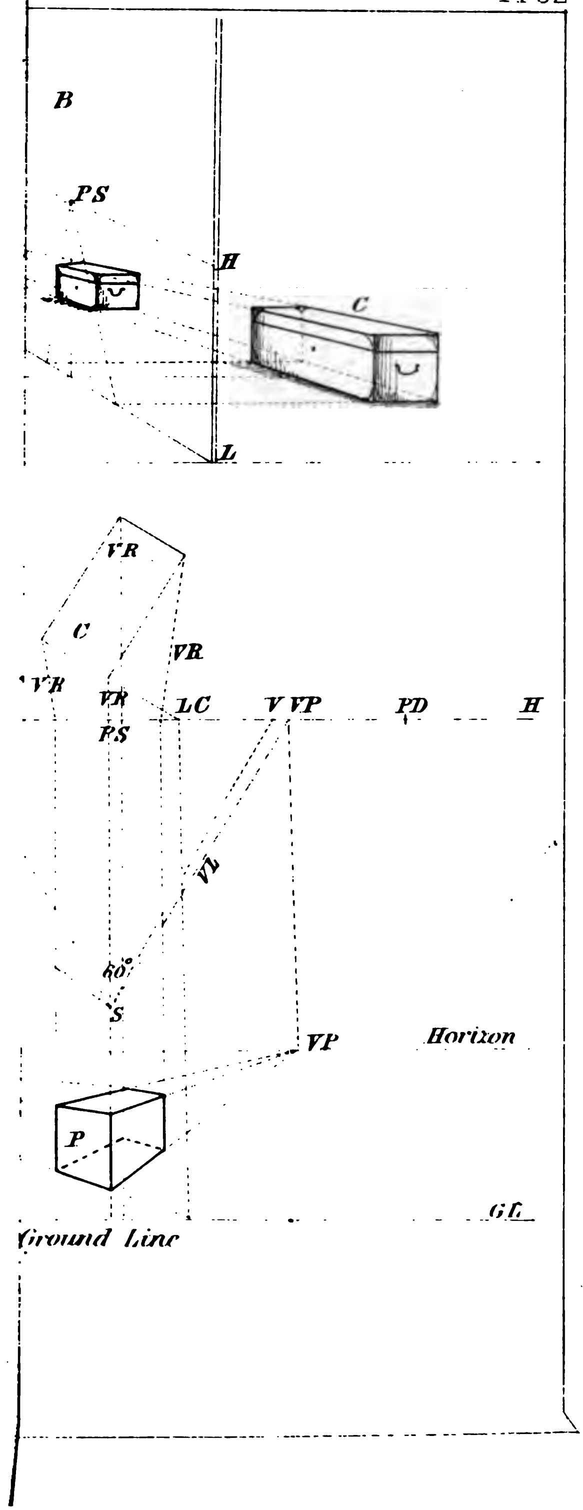
8
d
D
C
A
B
E
F


B
PS
H
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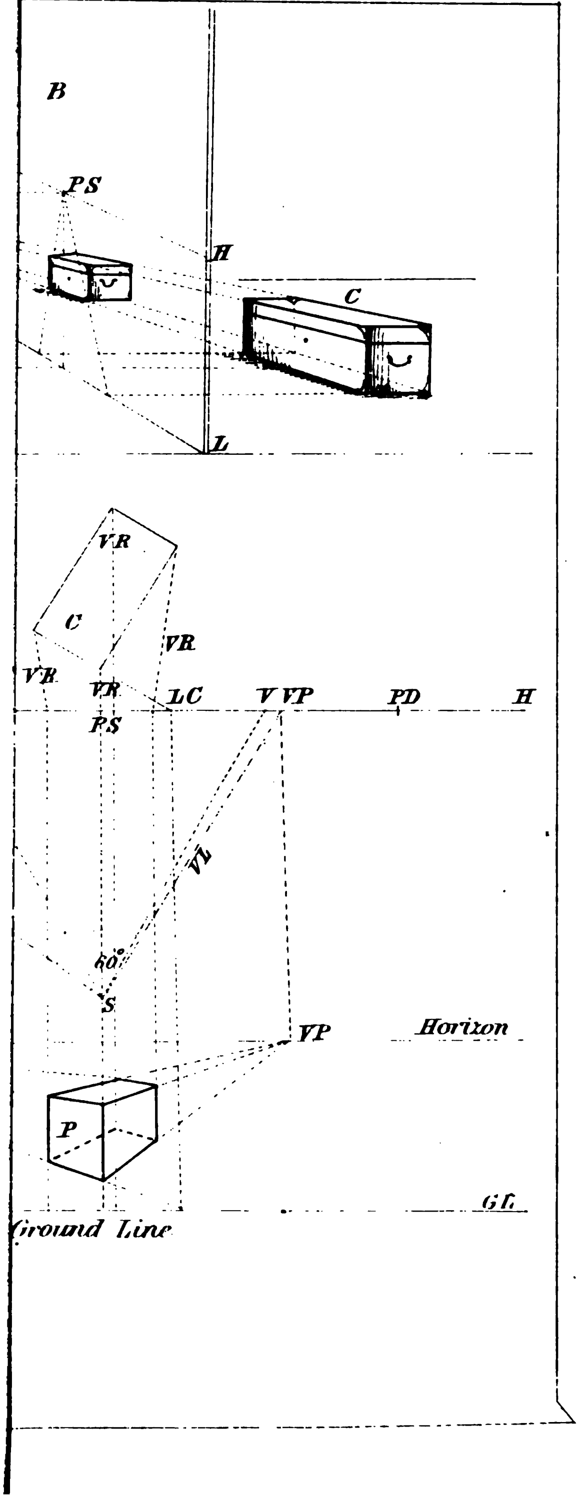
VR
C
VR
VR
VR
LC
V VP
PD
H
PS
VL
60°
S
VP
Horizon
P
GL
Ground Line

Pl. 52


B
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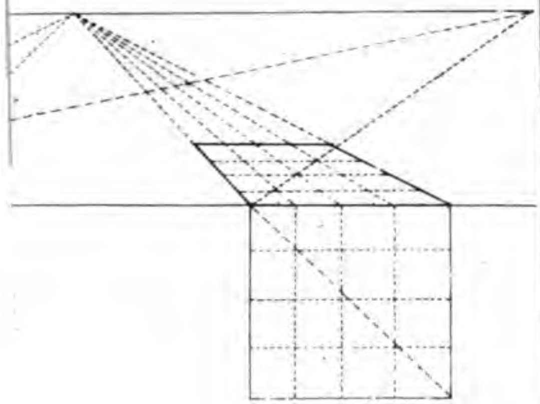
C
VR
VR
VR
LC
V VP
PD
H
PS
VL
60°
S
VP
Horizon
P
GL
Ground Line

G. 6



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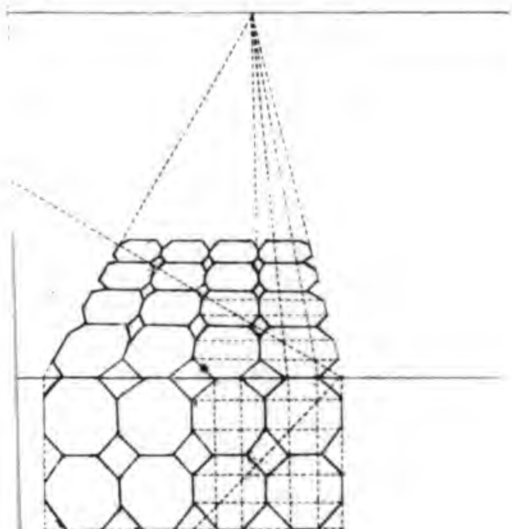

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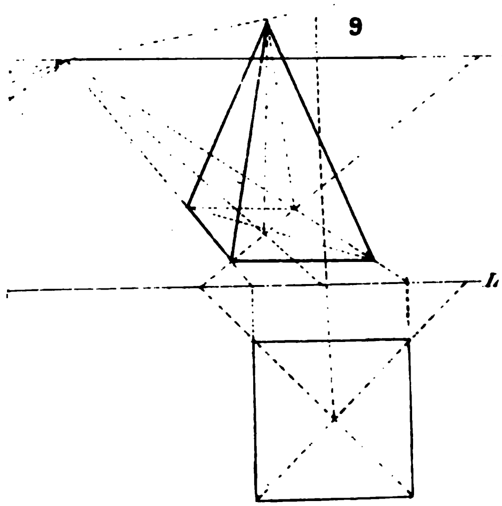

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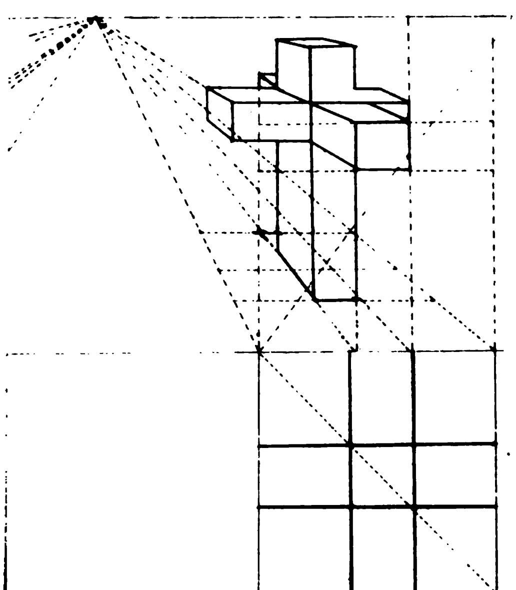
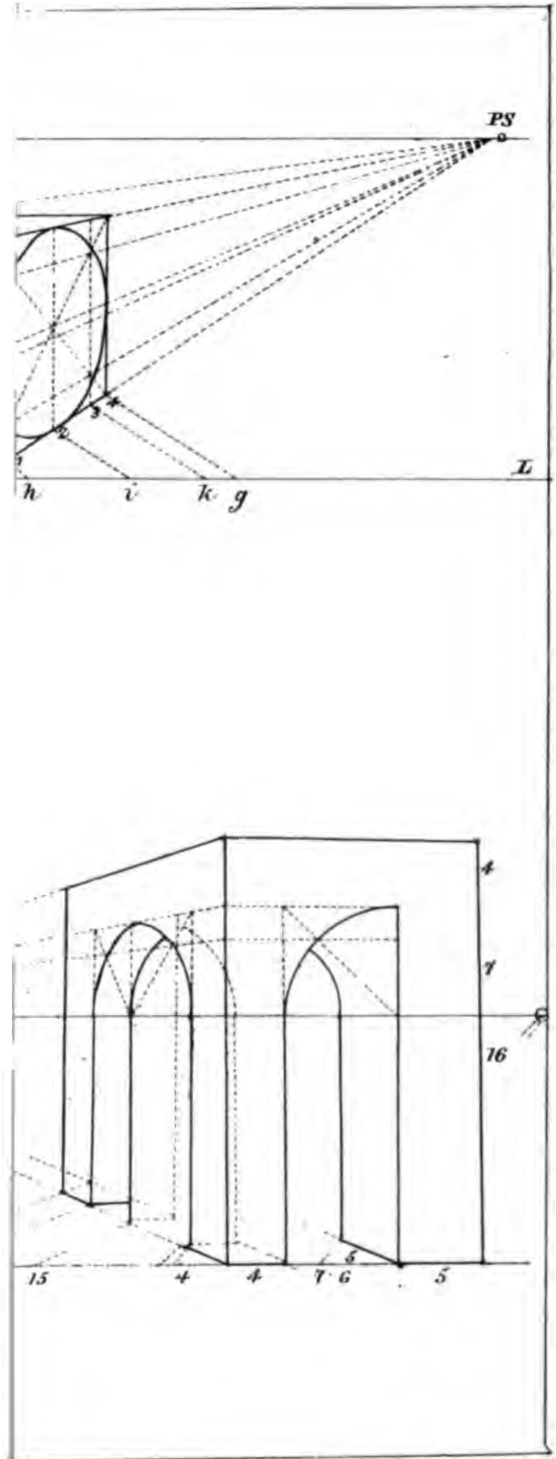
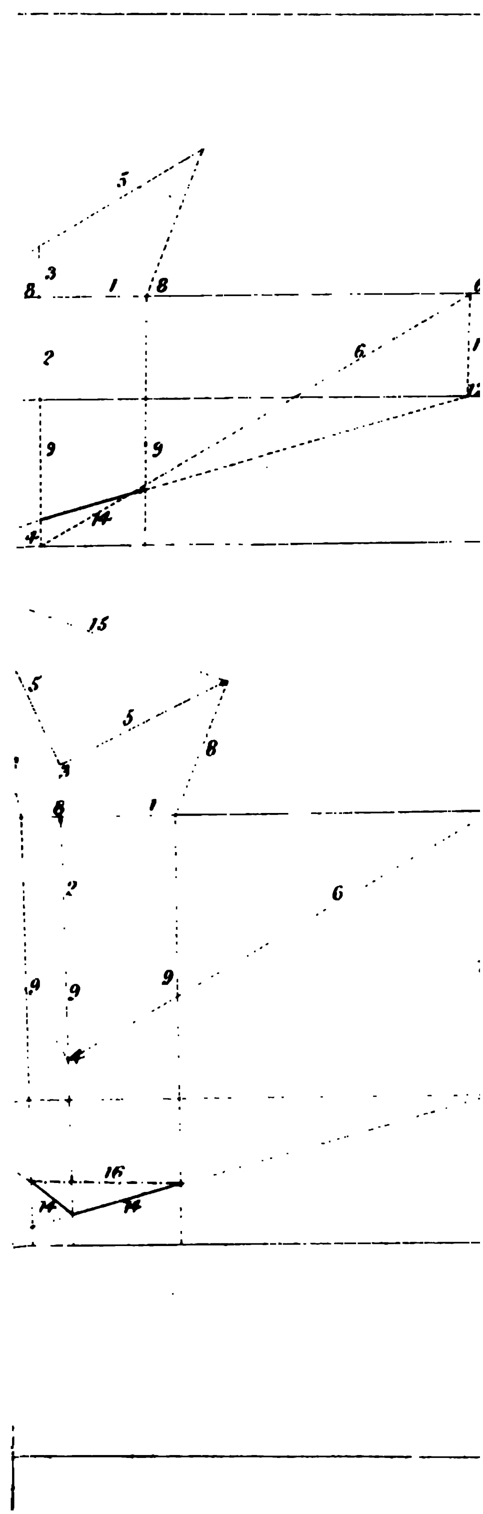


Pl. 56




5
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9
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15
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9
9
4
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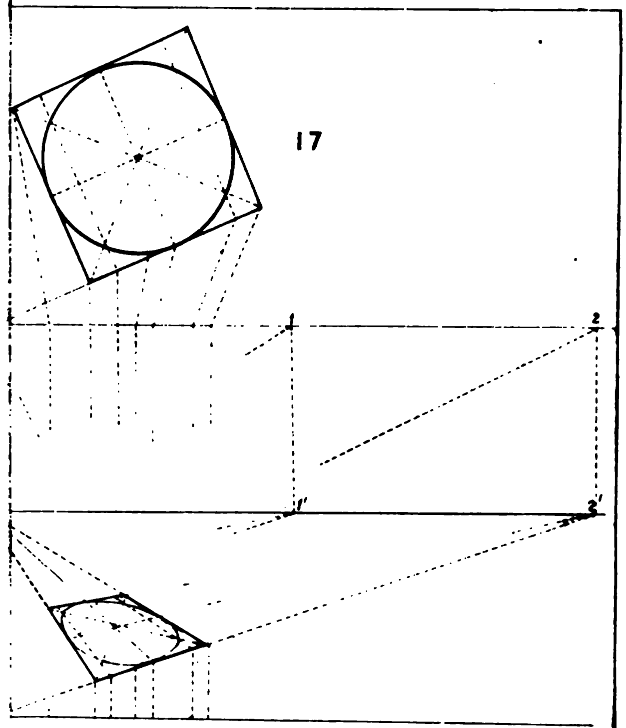
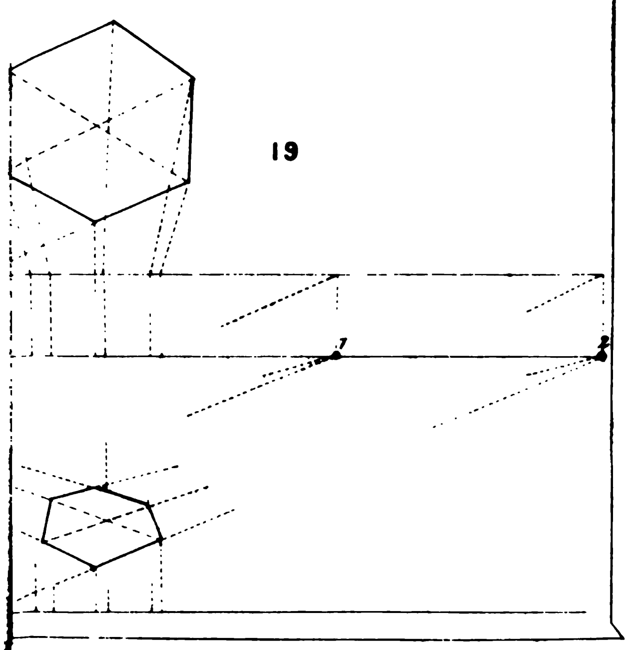
14
14

Pl. 58

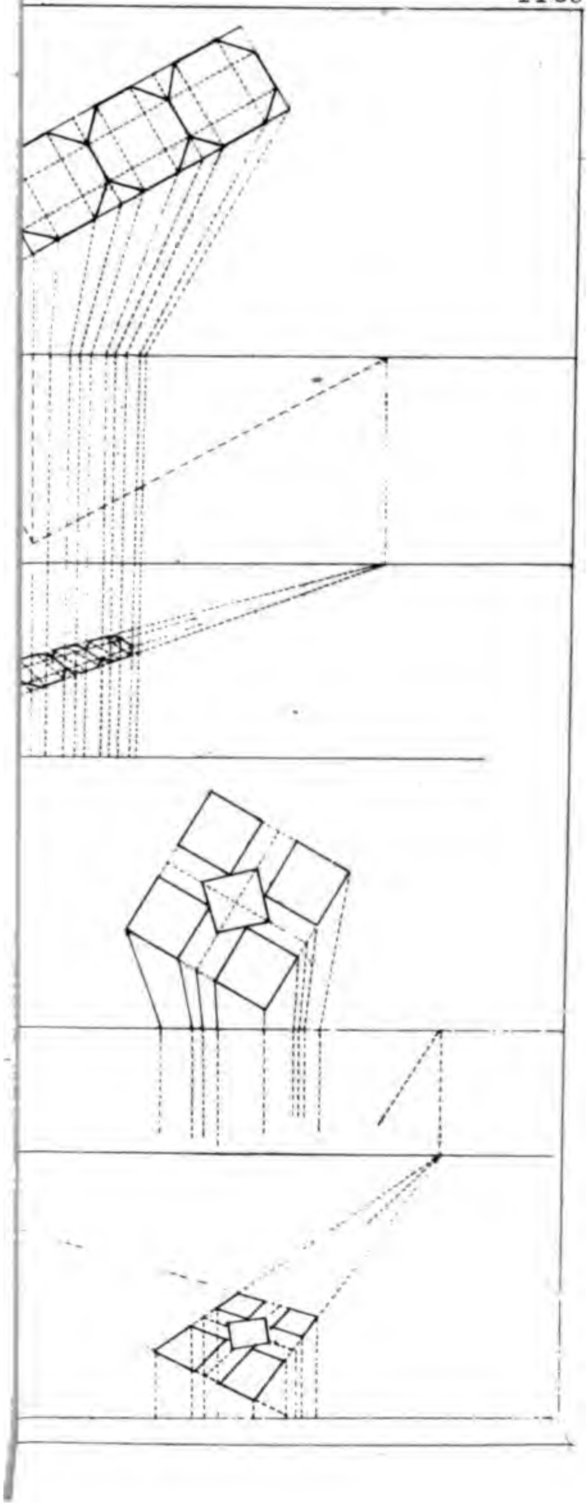
17



19



Pl 59



Pl 60

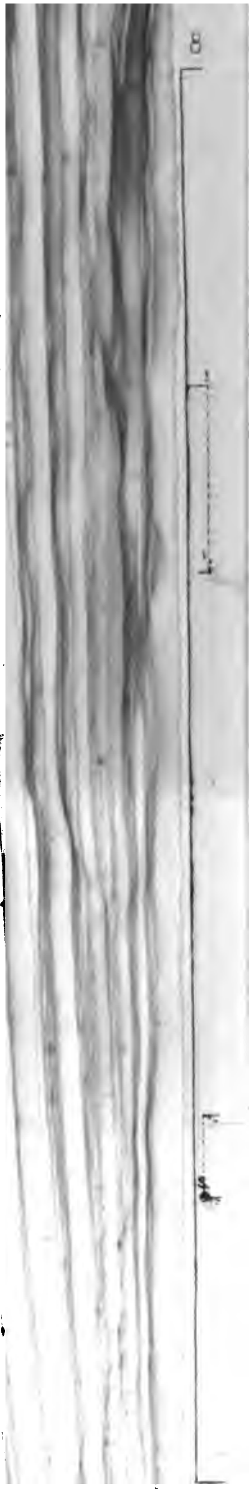

8

FIG. 22

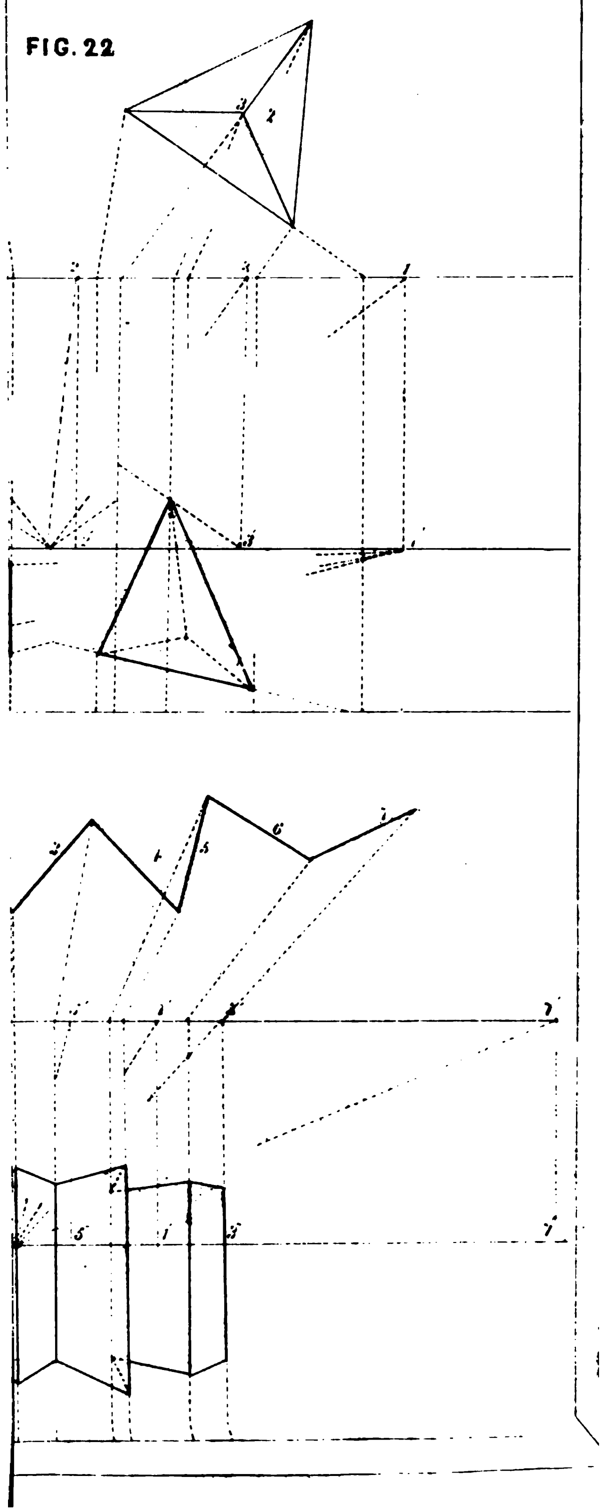
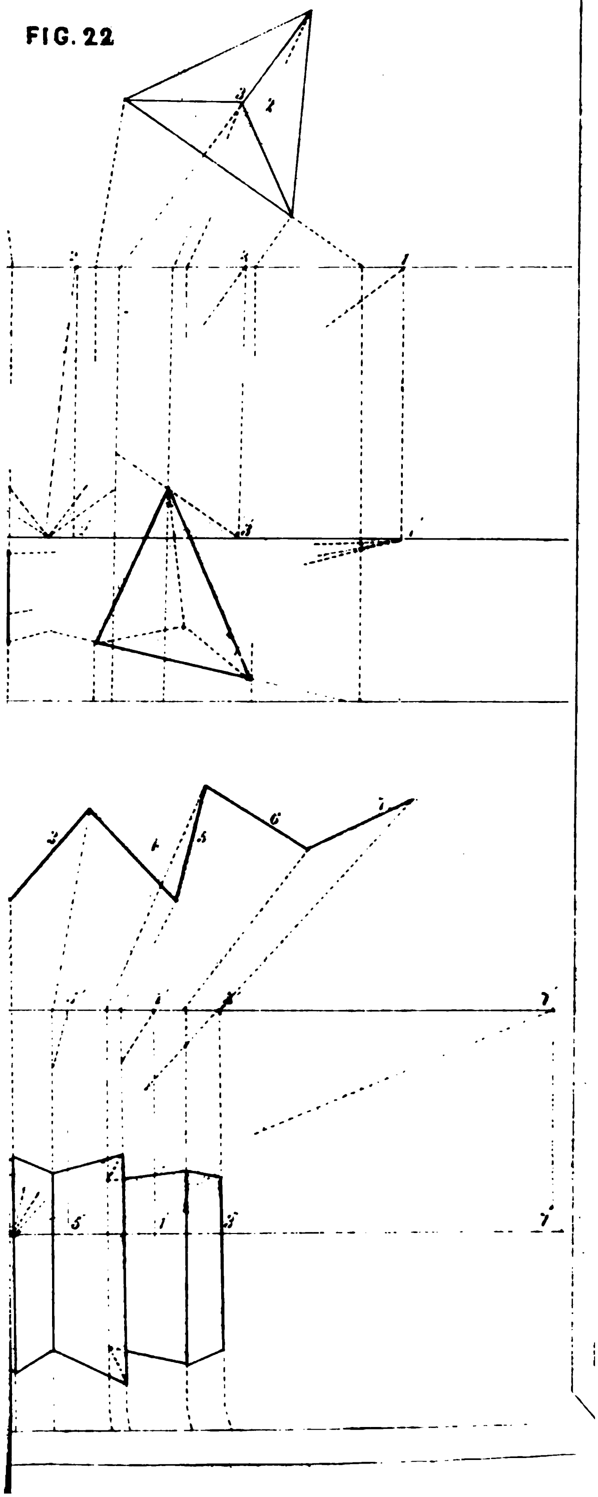


FIG. 22



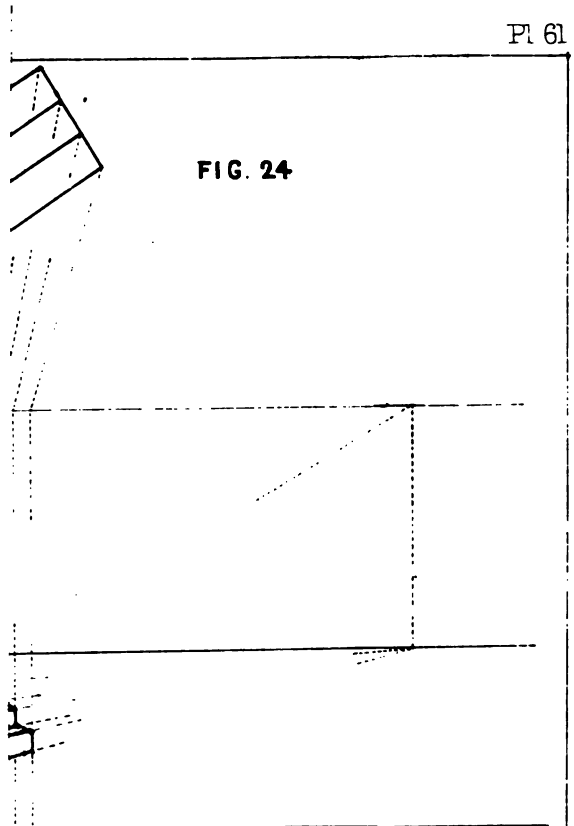
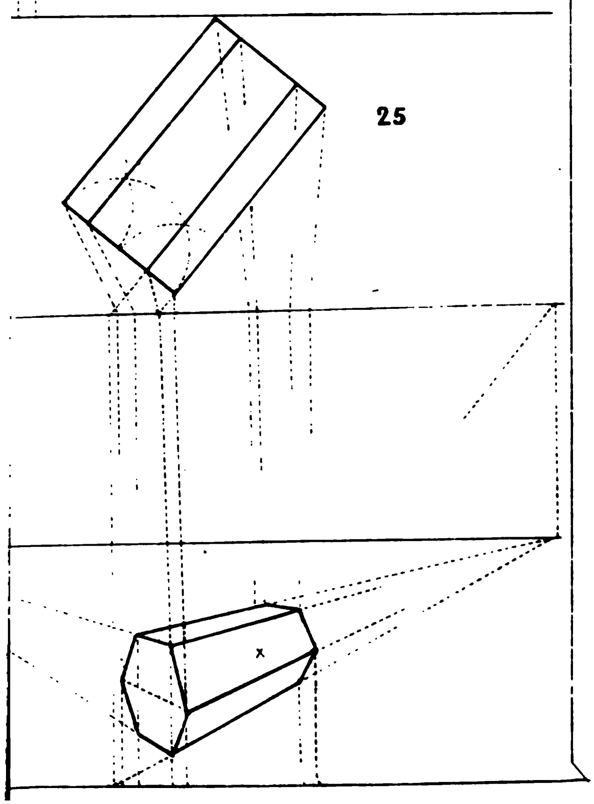


FIG. 24



25

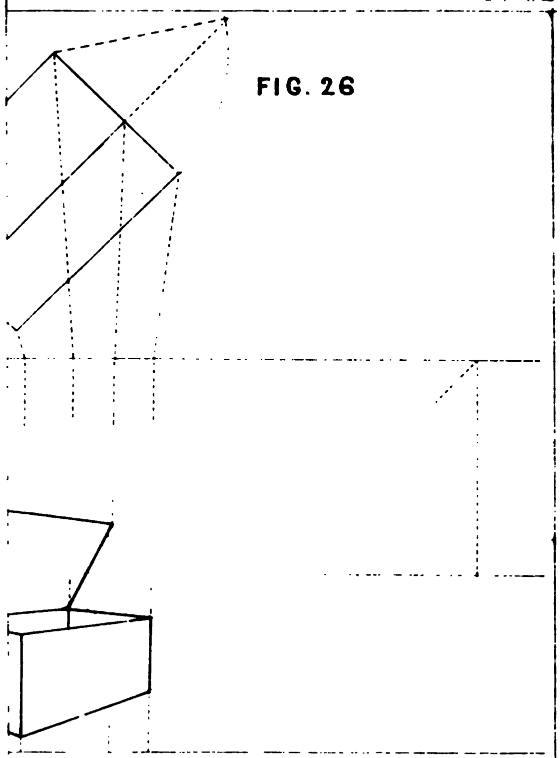
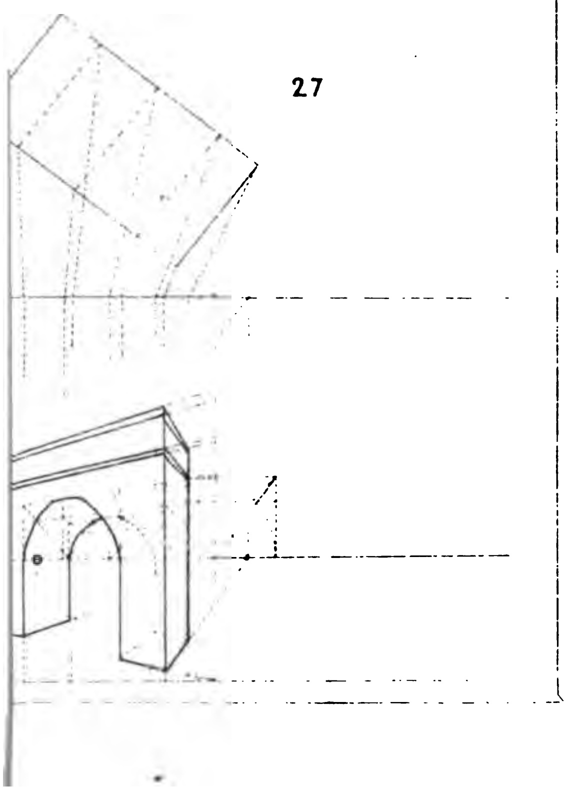
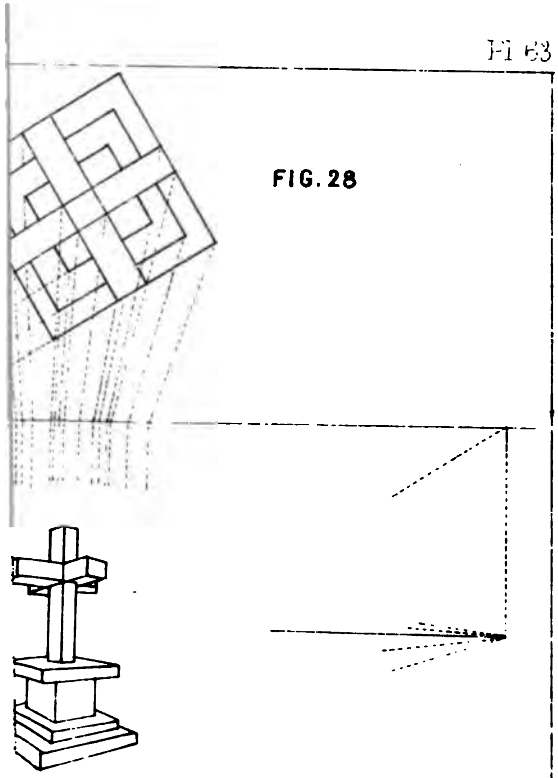


FIG. 26

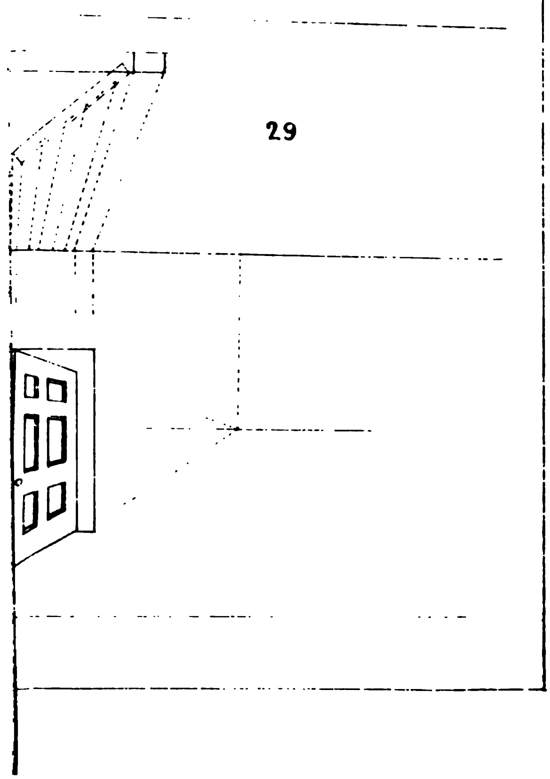


27

FIG. 28



29



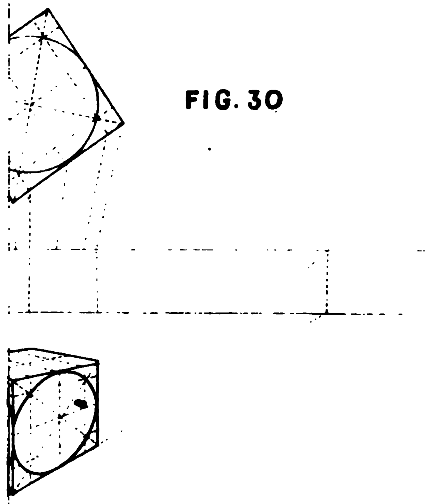
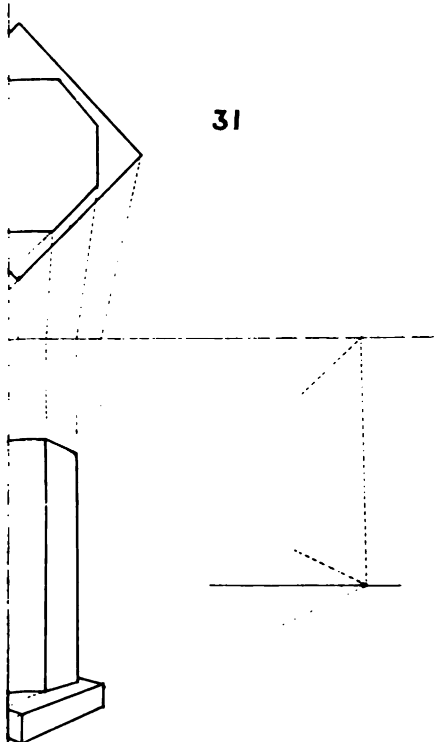
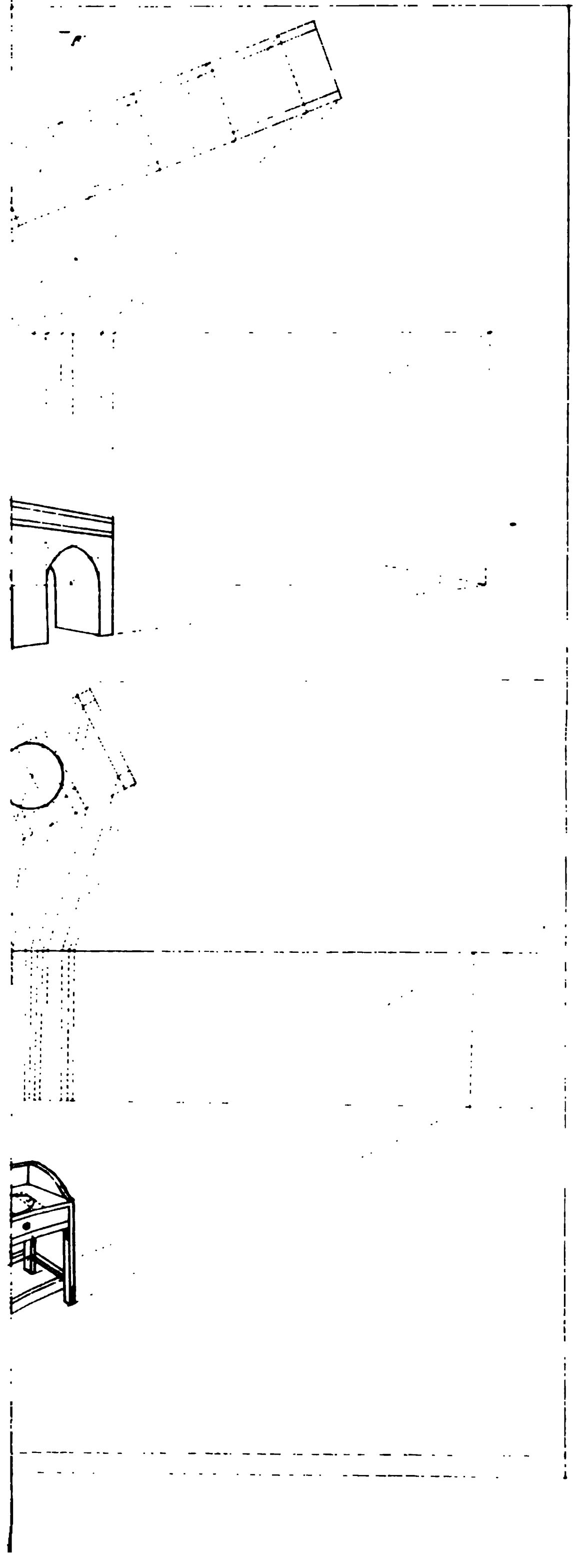


FIG. 30



31



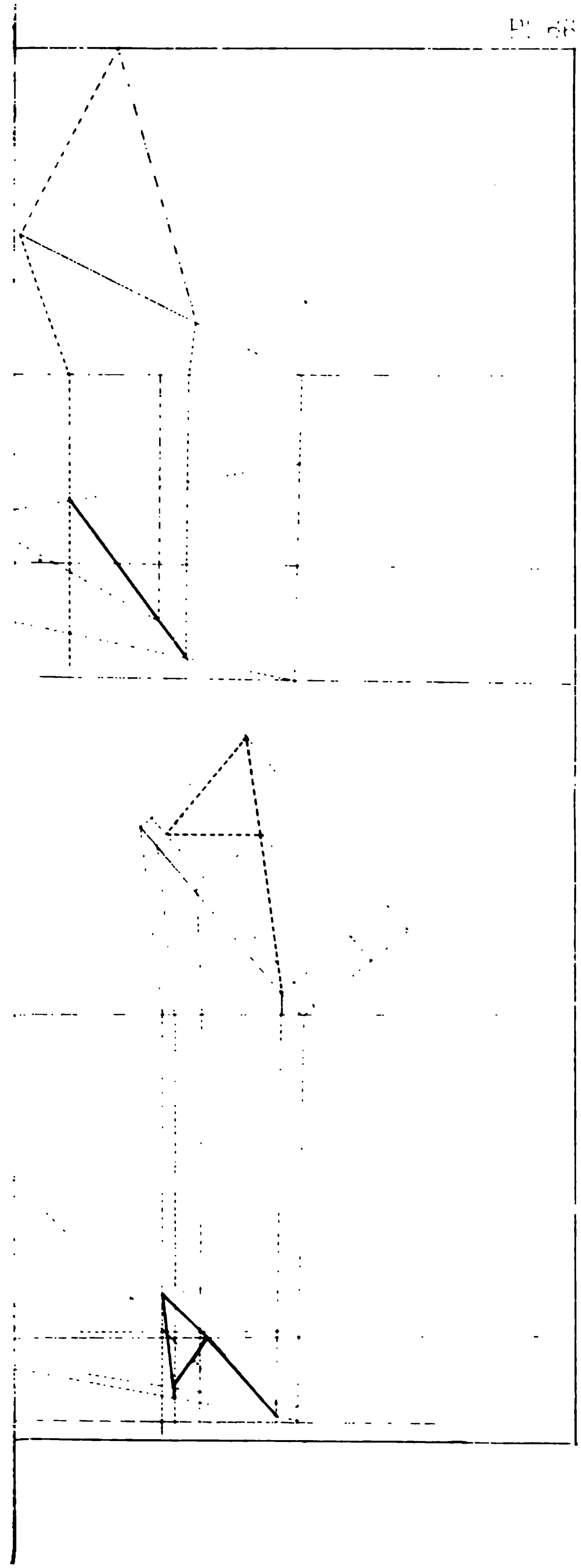
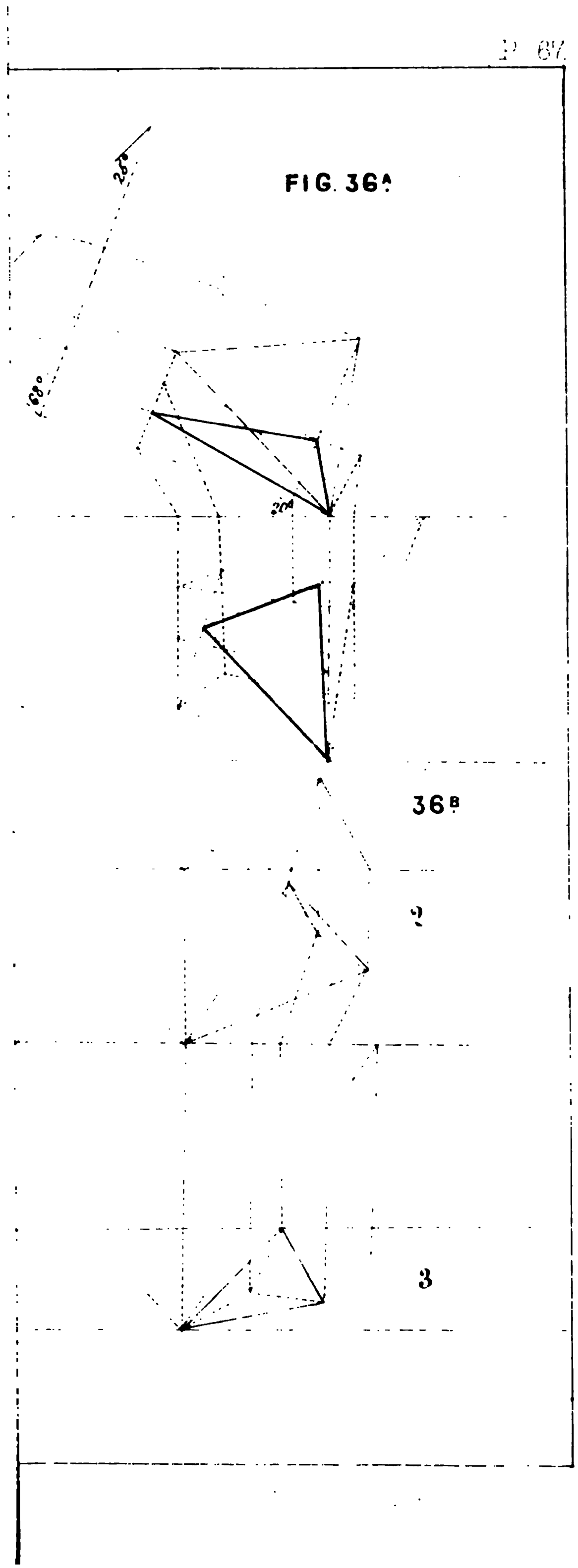


FIG. 36A

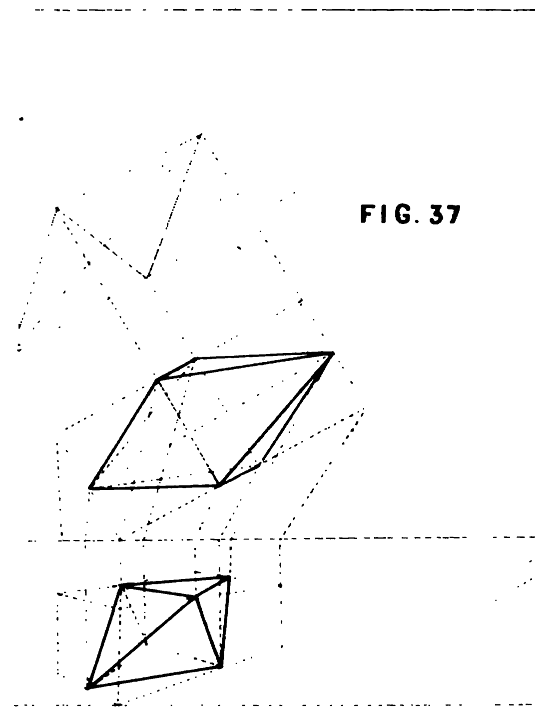


36B

2

3

FIG. 37



38